Math 216 - 1st Final Exam

Name _____

December 1, 2010

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (20 points) A 2-kg mass is attached to a spring with spring constant $24\,\text{N/m}$. The damping constant for the system is $8\,\text{N-sec/m}$. The mass starts at the equilibrium position with an initial speed of $2\,\text{m/sec}$ to the left. Is this mass-spring system underdamped, overdamped, or critically damped? Set up and solve the initial value problem that describes the displacement of the mass from equilibrium.

$$x = 0$$
(Equilibrium)
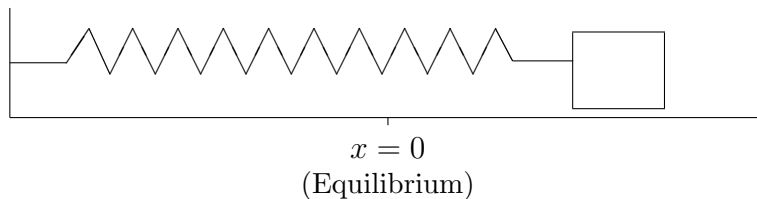
2. (16 points) Consider the equation $(x^2 + 1)y'' - 2xy' + 2y = 0$.

(a) Verify that $y_1(x) = (1 - x^2)$ and $y_2(x) = x$ are solutions.

(b) Use the Wronskian to show that y_1 and y_2 are linearly independent.

(c) Use what you've learned in parts (a) and (b) to find the solution of the IVP $(x^2 + 1)y'' - 2xy' + 2y = 0$; $y(1) = 1$, $y'(1) = -1$.

(d) Is your solution in part (c) unique? Explain.

3. (13 points) Test that the following equation is exact and then solve.

$$(2xe^{3y} + e^x)\,dx + (3x^2e^{3y} - y^2)\,dy = 0$$

4. (10 points) Find the orthogonal trajectories for the family of curves described by the equation $y = Cx^3$.

5. (20 points) Use variation of parameters to solve the following differential equation.

$$x'' - 2x' + x = \frac{e^t}{t^2}$$

6. (20 points) A 300-gallon tank is filled with 200 gallons of pure water. A spigot is opened above the tank, and a salt water solution containing $1\,\text{lb}$ of salt per gallon begins flowing into the tank at a rate of $6\,\text{gal/min}$. Simultaneously, a drain is opened at the bottom of the tank allowing the solution to leave the tank at a rate of $4\,\text{gal/min}$. How much salt will be in the solution at the precise moment when tank reaches its maximum capacity?

7. (10 points) Solve: $\quad y'' + 5y' + 6y = 0, \quad y(0) = y'(0) = 1$

8. (12 points) Determine the recursive formula for the Taylor method of order 2 for the initial value problem. Let $h = 0.1$ and use your formula to approximate $y(0.2)$.

$$y' = 4x^3 y, \quad y(0) = 5$$

9. (10 points) Solve: $\quad 7x'' + 3x' + 5x = 0$

10. (6 points) Referring to the problem above, suppose the equation was changed to

$$7x'' + 3x' + 5x = t \cos 2t.$$

Use your undetermined coefficients table to find the appropriate <u>form</u> of the particular solution for the nonhomogeneous equation. Do not solve for the undetermined coefficients.

11. (13 points) Solve: $\quad y' = xy^2 \cos x, \quad y(2\pi) = 1$