Math 216 - Test 2a

October 20, 2015

Name _____

Score _____

Show all work. Supply explanations where necessary.

1. (10 points) Solve: $y'' - 2y' - 15y = 0;\ y(0) = 3,\ y'(0) = -8$

2. (8 points) Find the general solution of $y^{(4)} - 3y'' = 0$.

3. (10 points) Solve the initial value problem.

$$x\,\frac{dy}{dx} = y + xe^{y/x},\ y(1) = 1$$

4. (8 points) Determine the recursive formulas for the Taylor method of order 2 for the IVP

$$\frac{dy}{dx} = x + y^2, \quad y(0) = 0.$$

5. (16 points) An object is launched from the ground into the air so that its velocity, in meters per second, at any time t (in seconds) satisfies the initial value problem

$$v' = -0.5v - 9.8, \quad v(0) = 60.$$

Determine the function that gives the height of the object at time t. Then estimate when the object will hit the ground.

6. (12 points) Consider the one-parameter family of curves described by

$$y = \frac{Cx}{1+x}.$$

Find the family of orthogonal trajectories.

7. (6 points) Consider the equation $xy'' - y' = 0,\ 0 < x < \infty$.

(a) Verify that $y_1(x) = 1$ and $y_2(x) = x^2$ are solutions.

(b) Use the Wronskian to show that y_1 and y_2 are linearly independent on $(0, \infty)$.

(c) Using your results from above, state the general solution of the ODE.

Math 216 - Test 2b

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Show all work. Supply explanations where necessary.

1. (10 points) Solve: $\dfrac{dy}{dx} = y(xy^3 - 1),\ y(0) = 1$

2. (10 points) Referring to the IVP above, use a Runge-Kutta method of order 4 or 5 to estimate the x-value at which $y(x) = 0.5$. Start by using stepsize $h = 0.1$, then decrease your stepsize until you believe your solution has at least 3 correct digits. Show enough "work" to be worth 10 points!

3. (10 points) Criminals in a boat are at the point $(1, 0)$ when the police shine a spotlight on them. The criminals evade the police by constantly moving counter-clockwise at a $45°$ angle from the light beam (which is following them). It turns out that the path of the criminals' boat satisfies the IVP

$$\frac{dy}{dx} = \frac{y/x + 1}{1 - y/x}, \quad y(1) = 0.$$

Solve the IVP. Then use the Implicit Equations Grapher (do a Google search) to graph the path.