Math 216 - 1st Final Exam

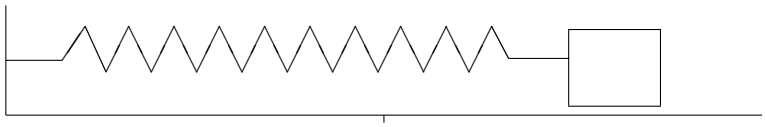
May 2, 2012

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) A 3-kg mass is attached to a spring with spring constant 3 N/m. The damping constant for the system is 6 N-sec/m. The mass is moved 1 m to the LEFT of equilibrium (compressing the spring) and pushed to the RIGHT at 1 m/sec. Set up and solve the initial value problem that describes the displacement of the mass from equilibrium. Is the mass-spring system underdamped, overdamped, or critically damped?



$x = 0$
(Equilibrium)

2. (10 points) Find the general solution of $y''' + 2y'' + 2y' = 0$.

3. (15 points) Solve: $\quad xy' + 2y = x^2, \quad y(1) = 1$

4. (15 points) One solution of the following equation is $y(x) = \sqrt{x}$.

$$4x^2y'' - (20x^2 + 4x)y' + (10x + 3)y = 0$$

Find the general solution. (Hint: Be sure to rewrite the equation in standard form.)

5. (12 points) Solve: $\quad y' = xy^{-3}e^x, \quad y(0) = 1$

6. (12 points) Show that the following equation is exact and solve.

$$(4xy^{1/2} + x^2 - 2x^{-1/2}y^{1/2})\,dx + (2y + x^2y^{-1/2} - 2x^{1/2}y^{-1/2})\,dy = 0$$

7. (10 points) Use Euler's method with a step size of $h = 0.5$ to approximate $y(3)$, where $y(x)$ is the solution of the initial value problem $y' = xy^2,\ y(2) = 1$.

8. (20 points) Solve.

$$\begin{array}{rcrcrcl} y' & - & 2y & + & z & = & 0, \\ z' & - & y & - & 2z & = & 0, \end{array} \qquad \begin{array}{rcl} y(0) & = & 1 \\ z(0) & = & 0 \end{array}$$

9. (20 points) Use undetermined coefficients to find the general solution of the following equation:

$$x'' - 4x' + 4x = 2 + 3e^t$$

10. (10 points) Find the orthogonal trajectories for the family of curves described by the equation $Cy^2 = x^3$.

11. (6 points) For $x > 0$, let $y_1(x) = \ln x^5$ and $y_2(x) = \ln x$. Compute the Wronskian of y_1 and y_2. Briefly explain why $y(x) = c_1 y_1(x) + c_2 y_2(x)$ cannot be the general solution of a 2nd-order, linear, homogeneous differential equation.

12. (4 points) What does it mean for two families of curves to be orthogonal trajectories of one another?

13. (4 points) Write a differential equation that would describe a mass-spring system that is overdamped. Do not solve your equation, but explain how you know the system is overdamped.