Math 216 - Quiz 1
February 5, 2014

Name key
Score

Show all work to receive full credit. Supply explanations when necessary. DO NOT USE A CALCULATOR FOR ANY PART OF THIS QUIZ.

1. (3 points) The following initial value problem does not have a closed-form solution that involves elementary functions. Nonetheless, the solution can be written in terms of a definite integral.

$$\frac{dy}{dx} = y\, e^{-x^2}, \quad y(0) = 2$$

Solve the IVP.

$$\frac{1}{y}\, dy = e^{-x^2} dx$$

$$\ln|y| = \int e^{-x^2} dx + C_1$$

$$|y| = e^{\int e^{-x^2} dx + C_1}$$

$$= C_2\, e^{\int e^{-x^2} dx}$$

$$y(x) = C_3\, e^{\int e^{-x^2} dx}$$

$$y(0) = 2$$

$$\Rightarrow$$

$$\boxed{y(x) = 2e^{\int_0^x e^{-t^2} dt}}$$

2. (3 points) Solve: $xy' = (1 - 2x^2)\tan y; \quad x > 0,\ y > 0$

$$\cot y\, dy = \frac{1-2x^2}{x} dx$$

$$\int \cot y\, dy = \int \left(\frac{1}{x} - 2x\right) dx$$

$$\ln|\sin y| = \ln x - x^2 + C_1$$

$$|\sin y| = e^{\ln x - x^2 + C_1}$$

$$|\sin y| = C_2\, x\, e^{-x^2}$$

$$\sin y = C_3\, x\, e^{-x^2}$$

$$\boxed{y(x) = \sin^{-1}\left(C_3\, x\, e^{-x^2}\right)}$$

3. (1 point) Use a direction field generator (see the link on the class website) to construct the direction field for the ODE $y' = 1 + x - y$. Use the direction field to guess the solution passing through $(0,0)$. SEE ATTACHED SHEET.

IT LOOKS LIKE THE SOLUTION THROUGH $(0,0)$ IS $y(x) = x$.

THIS IS EASY TO VERIFY:

IF $y = x$; THEN $y' = 1 = 1 + x - y$.

4. (3 points) Consider the differential equation $\frac{dy}{dx} = xy^{1/2}$, $y \geq 0$.

(a) Referring to our existence/uniqueness theorem, explain why we should not expect this ODE to have a unique solution passing through any point where $y = 0$.

$f(x,y) = xy^{1/2}$ IS CONTINUOUS FOR $0 \leq y < \infty$,

BUT $f_y(x,y) = \frac{x}{2\sqrt{y}}$ IS NOT DEFINED FOR $y = 0$.

BASED ON THE THEOREM, WE ARE NOT GUARANTEED A UNIQUE SOL'N THROUGH ANY POINT WHERE $y = 0$.

(b) Use a direction field generator to construct the direction field for the ODE. How many solutions pass through $(0,0)$? SEE ATTACHED SHEET.

2. ONE APPEARS TO BE $y(x) = 0$.

THE OTHER APPEARS TO HAVE A U-SHAPED GRAPH.

(c) Find the solutions passing through $(0,0)$.

$$y^{-1/2}\,dy = x\,dx$$

$$2y^{1/2} = \frac{1}{2}x^2 + C_1$$

$$y^{1/2} = \frac{x^2}{4} + C_2$$

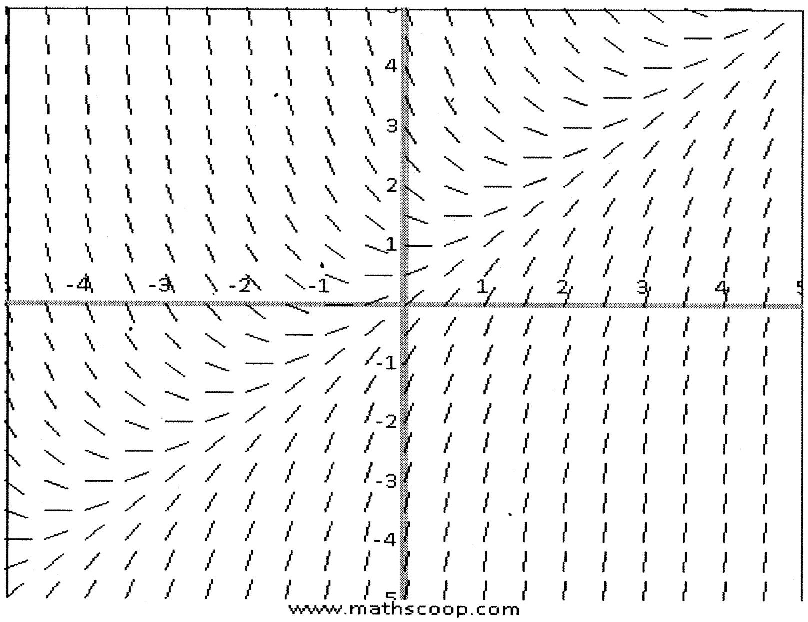
$$y(x) = \left(\frac{x^2}{4} + C_2\right)^2$$

$y(0) = 0 \Rightarrow C_2 = 0$

$$\boxed{y(x) = \frac{x^4}{16}}$$

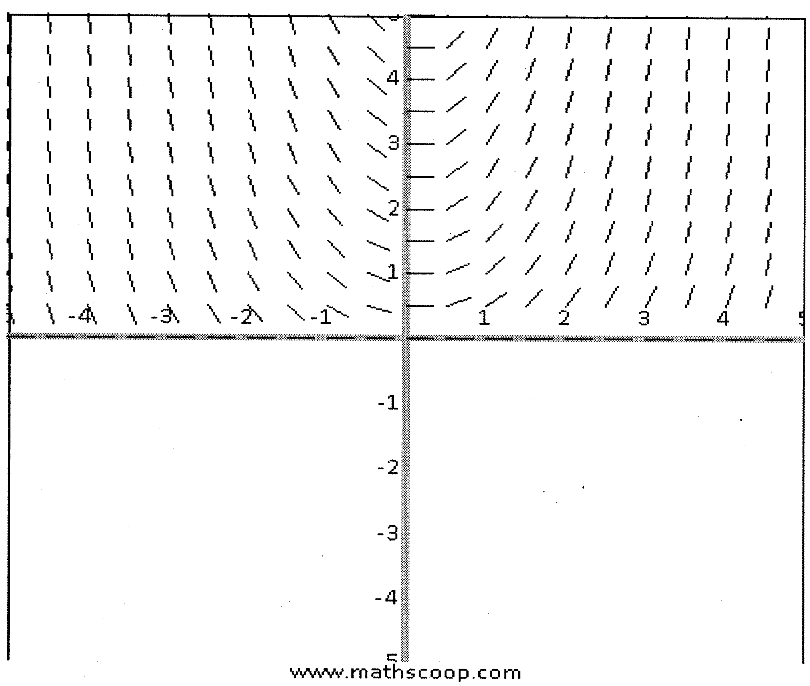
IT IS EASY TO SEE THAT THE CONSTANT FUNCTION $\boxed{y(x) = 0}$ IS ALSO A SOLUTION.

Problem #3



Equation : 1+x-y

Problem #4a



Equation : x*y^(1/2)