Math 216 - Quiz 2

February 12, 2014

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Solve: $\frac{dy}{dx} + 4y = x^2 e^{-4x}$, $y(0) = 2$

$$\mu(x) = e^{\int 4\,dx} = e^{4x}$$

$$y(x) = e^{-4x}\int x^2\,dx = e^{-4x}\left(\frac{x^3}{3} + c\right)$$

$$y(0) = 2 \Rightarrow \quad c = 2$$

$$\boxed{y(x) = e^{-4x}\left(\frac{x^3}{3} + 2\right)}$$

2. (2 points) Solve: $\underbrace{\left(ye^{xy} - \frac{1}{y}\right)}_{M(x,y)}\,dx + \underbrace{\left(xe^{xy} + \frac{x}{y^2}\right)}_{N(x,y)}\,dy = 0$

$$\frac{\partial M}{\partial y} = e^{xy} + yxe^{xy} + \frac{1}{y^2} = \frac{\partial N}{\partial x} \Rightarrow \text{EQUATION IS EXACT.}$$

$$\frac{\partial F}{\partial x} = ye^{xy} - \frac{1}{y} \Rightarrow F(x,y) = e^{xy} - \frac{x}{y} + g(y)$$

$$\frac{\partial F}{\partial y} = xe^{xy} + \frac{x}{y^2} \Rightarrow F(x,y) = e^{xy} - \frac{x}{y} + h(x)$$

$$F(x,y) = e^{xy} - \frac{x}{y} \Rightarrow \boxed{\text{SOLUTION IS } e^{xy} - \frac{x}{y} = C}$$

3. (3 points) Read problem #35 on page 54. The initial value problem described in that mixing problem is:

$$\frac{dA}{dt} = 1 - \frac{1}{100}A, \; A(0) = 5.$$

Solve the IVP. Then find the amount of salt in the tank after 10 minutes.

$$\frac{dA}{dt} + \frac{1}{100}A = 1$$

$$\mu(t) = e^{\int \frac{1}{100}\,dt} = e^{t/100}$$

$$A(t) = e^{-t/100}\int e^{t/100}(1)\,dt$$

$$= e^{-t/100}\left(100\,e^{t/100} + C\right)$$

$$A(0) = 5 \Rightarrow 100 + C = 5 \Rightarrow C = -95$$

$$A(t) = 100 - 95e^{-t/100}$$

$$A(10) = 100 - 95e^{-1/10} \approx 14.04$$

14.04 kg

4. (3 points) The solution of the following IVP must be written in terms of a definite integral (with bounds from 2 to x).

$$\frac{dy}{dx} = 1 - 2xy, \; y(2) = 1$$

(a) Solve the IVP.

$$e^{x^2} y(x) = C + \int_2^x e^{t^2}\, dt$$

$$\frac{dy}{dx} + 2xy = 1$$

$$\mu(x) = e^{\int 2x\, dx} = e^{x^2}$$

$$\mu(x)\, y(x) = \int e^{x^2}(1)\, dx$$

$$y(2) = 1 \Rightarrow$$

$$e^4(1) = C \Rightarrow C = e^4$$

$$y(x) = e^{-x^2}\left(e^4 + \int_2^x e^{t^2}\, dt\right)$$

(b) Use the numerical integration feature on your calculator or computer algebra system to compute $y(3)$.

$$y(3) = e^{-9}\left(e^4 + \int_2^3 e^{t^2}\, dt\right)$$

$$\approx 0.18298$$

(c) Use Euler's method with $h = 0.1$ to approximate $y(3)$.

Using the program on my TI-84,

$$y(3) \approx 0.1783584913$$

↑ Showing everything the calculator displayed.