

**Math 216 - Quiz 3**

February 26, 2014

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Solve:  $(xy - 2) dx + (x^2 - xy) dy = 0$

$\underbrace{(xy - 2)}_{M(x,y)} \quad \underbrace{(x^2 - xy)}_{N(x,y)}$

$$\frac{\partial M}{\partial y} = x \quad \frac{\partial N}{\partial x} = 2x - y$$

$$\frac{x - 2x + y}{x(x-y)} = \frac{y-x}{x(x-y)} = \frac{-1}{x} = g(x)$$

$$\mu(x) = e^{\int -\frac{1}{x} dx} = e^{-\ln|x|} = \frac{1}{|x|} = \frac{1}{x}, \quad \text{Assuming } x > 0$$

MULT BY  $\frac{1}{x}$  ...

$$\left(y - \frac{2}{x}\right) dx + (x - y) dy = 0$$

$$\frac{\partial F}{\partial x} = y - \frac{2}{x}$$

$$\Rightarrow F(x,y) = xy - 2\ln x + g(y)$$

$$\frac{\partial F}{\partial y} = x - y$$

$$\Rightarrow F(x,y) = xy - \frac{y^2}{2} + h(x)$$

$$F(x,y) = xy - 2\ln x - \frac{y^2}{2}$$

Sol'n is

$$xy - 2\ln x - \frac{y^2}{2} = C, \quad x > 0$$

2. (3 points) Solve:  $x^2 y' = 2y^2 - x^2$

$$y' = \frac{2y^2}{x^2} - 1$$

$$u = \frac{y}{x} \Rightarrow ux = y$$

$$\Rightarrow u + x \frac{du}{dx} = \frac{dy}{dx}$$

$$u + x \frac{du}{dx} = 2u^2 - 1$$

$$x \frac{du}{dx} = 2u^2 - u - 1$$

$$\frac{1}{2u^2 - u - 1} du = \frac{1}{x} dx$$

$$\frac{1}{(2u+1)(u-1)} du = \frac{1}{x} dx$$

PFD:

$$\frac{1}{(2u+1)(u-1)} = \frac{1}{3(u-1)} - \frac{2}{3(2u+1)}$$

$$\int \frac{1}{3(u-1)} - \frac{2}{3(2u+1)} du = \int \frac{1}{x} dx$$

$$3 \left( \frac{1}{3} \ln|u-1| - \frac{1}{3} \ln|2u+1| \right) = 3(\ln|x| + C)$$

$$\ln \left| \frac{u-1}{2u+1} \right| = \ln|x|^3 + C$$

$$\frac{y/x - 1}{2y/x + 1} = Cx^3 \Rightarrow \frac{y-x}{2y+x} = Cx^3$$

3. (4 points) Solve:  $xy' + y = x^4y^3$

$$y' + \frac{1}{x}y = x^3y^3$$

$$y^{-3}y' + \frac{1}{x}y^{-2} = x^3$$

$$u = y^{-2}$$

$$u' = -2y^{-3}y'$$

$$-\frac{1}{2}u' = y^{-3}y'$$

$$-\frac{1}{2}u' + \frac{1}{x}u = x^3$$

$$u' - \frac{2}{x}u = -2x^3$$

$$\mu(x) = e^{\int -\frac{2}{x} dx} = e^{-2 \ln|x|} = \frac{1}{x^2}$$

$$u(x) = x^2 \int \frac{1}{x^2} (-2x^3) dx$$

$$= x^2 \int -2x dx$$

$$= x^2 (-x^2 + C)$$

$$u(x) = Cx^2 - x^4$$

$$y^{-2} = Cx^2 - x^4$$

$$y(x) = \frac{1}{\sqrt{Cx^2 - x^4}}$$