

Math 216 - Quiz 4

March 5, 2014

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Solve: $x^2 \frac{dy}{dx} - 2xy = 3y^4$, $y(1) = \frac{1}{2}$

$$y^{-4} \frac{dy}{dx} - \frac{2}{x} y^{-3} = \frac{3}{x^2}$$

$$u = y^{-3}$$

$$\frac{du}{dx} = -3y^{-4} \frac{dy}{dx}$$

$$\frac{du}{dx} + \frac{6}{x} u = -\frac{9}{x^2}$$

$$\mu(x) = e^{\int \frac{6}{x} dx} = e^{6 \ln|x|} = x^6$$

$$u(x) = x^{-6} \int -9x^4 dx = x^{-6} \left(-\frac{9}{5} x^5 + C \right)$$

$$u(x) = \frac{-9}{5x} + \frac{C}{x^6}$$

$$y(x) = \frac{1}{\sqrt[3]{\frac{-9}{5x} + \frac{C}{x^6}}}$$

$$y(1) = \frac{1}{2} = \frac{1}{\sqrt[3]{\frac{-9}{5} + C}} \Rightarrow C = \frac{49}{5}$$

$$y(x) = \frac{1}{\sqrt[3]{\frac{-9}{5x} + \frac{49}{5x^6}}}$$

2. (3 points) Find the orthogonal trajectories for the one-parameter family of curves described by $y = \frac{1}{x+c}$.

$$(x+c)y = 1$$

$$xy + cy = 1$$

$$c = \frac{1-xy}{y}$$

$$\frac{y(-y - xy') - (1-xy)y'}{y^2} = 0$$

$$\frac{-y^2 - xyy' - y' + xyy'}{y^2} = 0$$

$$\Rightarrow y' = -y^2$$

ORTHO TRAJ'S ...

$$y' = \frac{1}{y^2}$$

$$y^2 dy = dx$$

$$\int y^2 dy = \int dx$$

$$\frac{1}{3} y^3 = x + C_1$$

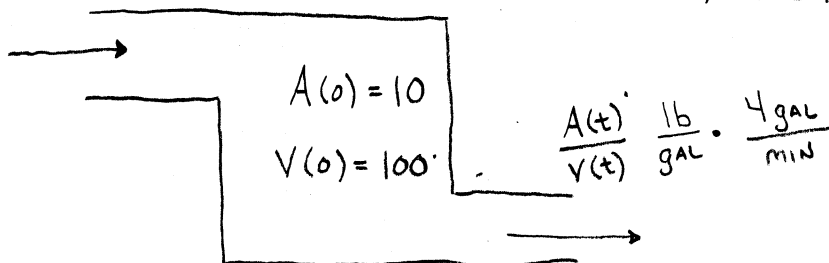
$$y = \sqrt[3]{3x + C_2}$$

3. (4 points) A large tank is partially filled with 100 gallons of fluid in which 10 lbs of salt is dissolved. Brine containing $\frac{1}{2}$ lb of salt per gallon is pumped into the tank at a rate of 6 gal/min. The well-mixed solution is then pumped out at a rate of 4 gal/min. Find the amount of salt in the tank after 30 minutes.

$$\frac{1}{2} \frac{\text{lb}}{\text{gal}} \cdot \frac{6 \text{ gal}}{\text{min}} = 3 \text{ lb/min}$$

$$V(t) = 100 + 2t$$

Adding 2 gal/min



$$\frac{dA}{dt} = 3 - \frac{4A}{100+2t}, \quad A(0) = 10$$

$$\frac{dA}{dt} + \frac{4}{100+2t} A = 3$$

$$\mu(t) = e^{\int \frac{4}{100+2t} dt} = e^{\frac{4}{2} \ln |100+2t|} = (100+2t)^2$$

$$A = (100+2t)^{-2} \int 3(100+2t)^2 dt$$

$$= (100+2t)^{-2} \left[\frac{1}{2} (100+2t)^3 + C \right]$$

$$= 50 + t + C(100+2t)^{-2}$$

$$A(0) = 10 \Rightarrow 50 + \frac{C}{100^2} = 10$$

$$\Rightarrow C = -400000$$

$$A(t) = 50 + t - 400000(100+2t)^{-2}$$

$$A(30) = 64.375 \text{ lb}$$