Math 216 - Quiz 6

April 9, 2014

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Solve: $4y'' + 4y' + 17y = 0; \quad y(0) = -1, y'(0) = 2$

$4r^2 + 4r + 17 = 0$

$r = \dfrac{-4 \pm \sqrt{16 - 4(4)(17)}}{8}$

$r = -\frac{1}{2} \pm 2i$

$y(x) = e^{-x/2}(c_1 \cos 2x + c_2 \sin 2x)$

$y(0) = -1 \Rightarrow c_1 = -1$

$y'(x) = -\frac{1}{2}e^{-x/2}(-\cos 2x + c_2 \sin 2x) + e^{-x/2}(2\sin 2x + 2c_2 \cos 2x)$

$y'(0) = 2 \Rightarrow \frac{1}{2} + 2c_2 = 2 \Rightarrow c_2 = \frac{3}{4}$

$$y(x) = e^{-x/2}\left(-\cos 2x + \frac{3}{4}\sin 2x\right)$$

2. (3 points) Solve: $y''' + 3y'' - 4y = 0$

$r^3 + 3r^2 - 4 = 0$

$(r-1)(r+2)^2 = 0$

$r = 1, \; r = -2, r = -2$

$$y(x) = c_1 e^{x} + c_2 e^{-2x} + c_3 x e^{-2x}$$

3. (4 points) Solve the Cauchy-Euler equation: $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = 0$

Let $x = e^t$.

This reduces the equation to

$\dfrac{d^2y}{dt^2} - 3\dfrac{dy}{dt} - 4y = 0$

$r^2 - 3r - 4 = 0$

$(r-4)(r+1) = 0$

$r = 4, \; r = -1$

$y(t) = c_1 e^{4t} + c_2 e^{-t}$

$$y(x) = c_1 x^4 + c_2 x^{-1}$$