Math 216 - Quiz 7
April 16, 2014

Name key
Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Solve: $y'' - 2y' + 2y = e^{2x}(\cos x - 3\sin x)$

Homo eq: $y'' - 2y' + 2y = 0$

$r^2 - 2r + 2 = 0$

$(r-1)^2 = -1$

$r - 1 = \pm i$

$r = 1 \pm i$

$\alpha = 1, \beta = 1$

$y_h(x) = c_1 e^x \cos x + c_2 e^x \sin x$

Nonhomo eq: $g(x) = e^{2x}(\cos x - 3\sin x)$

$y_p(x) = e^{2x}(A\cos x + B\sin x)$

$y_p'(x) = 2e^{2x}(A\cos x + B\sin x) + e^{2x}(-A\sin x + B\cos x)$

$y_p''(x) = 4e^{2x}(A\cos x + B\sin x) + 4e^{2x}(-A\sin x + B\cos x) + e^{2x}(-A\cos x - B\sin x)$

$y_p''(x) - 2y_p'(x) + 2y_p(x)$

$= (3e^{2x}A + 4e^{2x}B - 4e^{2x}A - 2e^{2x}B + 2e^{2x}A)\cos x$

$+ (3e^{2x}B - 4e^{2x}A - 4e^{2x}B + 2e^{2x}A + 2e^{2x}B)\sin x$

$= e^{2x}(\cos x - 3\sin x)$

$A + 2B = 1$

$B - 2A = -3$

$5A = 7 \Rightarrow A = \frac{7}{5}$

$B = -\frac{1}{5}$

$y_p(x) = e^{2x}\left(\frac{7}{5}\cos x - \frac{1}{5}\sin x\right)$

$y(x) = c_1 e^x \cos x + c_2 e^x \sin x + \frac{7}{5}e^{2x}\cos x - \frac{1}{5}e^{2x}\sin x$

2. (6 points) Solve: $y'' + 4y' + 4y = (3+x)e^{-2x};\ y(0) = 2, y'(0) = 5$

Homo eq: $y'' + 4y' + 4y = 0$

$r^2 + 4r + 4 = 0$

$(r+2)^2 = 0$

$r = -2, r = -2$

$y_h(x) = c_1 e^{-2x} + c_2 x e^{-2x}$

Nonhomo eq: $g(x) = (3+x)e^{-2x}$

$y_p(x) = x^2(Ax+B)e^{-2x}$

$y_p(x) = (Ax^3 + Bx^2)e^{-2x}$

$y_p'(x) = -2(Ax^3 + Bx^2)e^{-2x} + (3Ax^2 + 2Bx)e^{-2x}$

$y_p''(x) = 4(Ax^3 + Bx^2)e^{-2x} - 2(3Ax^2 + 2Bx)e^{-2x} - 2(3Ax^2 + 2Bx)e^{-2x} + (6Ax + 2B)e^{-2x}$

$y_p'' + 4y_p' + 4y_p = (3+x)e^{-2x}$

$\Downarrow$

$4Ax^3 + 4Bx^2 - 6Ax^2 - 4Bx - 6Ax^2 - 4Bx + 6Ax + 2B - 8Ax^3 - 8Bx^2 + 12Ax^2 + 8Bx + 4Ax^3 + 4Bx^2 = (3+x)$

$6A = 1 \quad A = 1/6$

$2B = 3 \quad B = 3/2$

$y_p(x) = \left(\frac{1}{6}x^3 + \frac{3}{2}x^2\right)e^{-2x}$

$y(x) = c_1 e^{-2x} + c_2 x e^{-2x} + \left(\frac{1}{6}x^3 + \frac{3}{2}x^2\right)e^{-2x}$

$y(0) = 2 \Rightarrow c_1 = 2$

$y'(x) = -2\left(c_1 + c_2 x + \frac{1}{6}x^3 + \frac{3}{2}x^2\right)e^{-2x} + \left(c_2 + \frac{3}{6}x^2 + 3x\right)e^{-2x}$

$y'(0) = 5 \Rightarrow -2c_1 + c_2 = 5$

$c_2 = 9$

$$y(x) = \left(2 + 9x + \frac{3}{2}x^2 + \frac{1}{6}x^3\right)e^{-2x}$$