Bobo Numbers, Bobbers, and Bears—Experiences in Undergraduate Research

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Undergraduate research experiences are well known for their effects on student learning and engagement. The presenter has mentored community college students whose research has won awards and even been published. In this presentation, you will learn some things about Bobo numbers, falling ladders and floating bobbers, former Chicago Bears head coach Lovie Smith, and student research, in general.

In this presentation, we'll discuss:

- The value of undergraduate research
- Advice and best practices for students in the first two years
- Some sources and ideas for research projects
- Some successful research projects

Undergraduate research in the first two years must be real research!

- It must involve original contributions and public dissemination.
- It need not make a great impact.
- Its value is not in research productivity.

Why do research in the first two years?

Well-documented positive effects on students:

- High impact on learning and engagement
- Improves attitude and self confidence
- Encourages life-long learning
- Leads to higher-order thinking, application, and integration
- More likely to complete undergraduate degree
- More likely to consider graduate school

Positive effects on faculty:

- Informs teaching
- Good pedagogy
- Brings fresh perspectives
- Provides variety and excitement
- Provides greater sense of purpose

Positive effects on the institution:

- Creates long-term connections between students and institution

- Affects college reputation

- Useful recruiting tool

Problems with undergraduate research in the 1st two years:

- Students lack necessary skills
- Students lack time and motivation
- Faculty lack time and motivation
- Institutional support is inadequate
- Its hard to find good problems
- Students own ideas often lack clarity and focus

Some sources of research projects:

- Many journal articles pose follow-up questions.

- Problems in textbooks can lead to Mythbusters-type experiments.

- Search the Online Encyclopedia of Integer Sequences.

- Exercises in textbooks can lead to deeper questions.

Some advice of my own...

- Aim low! Keep it simple.
- Emphasize originality over significance.
- Expect to commit lots of time.
- Expect to provide lots of support.
- Do not expect your student to be able to formulate a good question.
- Seemingly insignificant problems may lead somewhere.
- Do not expect your student to understand proof or Taylor's theorem or the value of inequalities or numerical methods or...

Samples of successful UR

In 2010, Prairie State College began to participate in a regional STEM competition. The math and chemistry departments have been especially successful. Here are brief descriptions of four math projects I have mentored.

Project 1: Lovie Smith and Separating Hyperplanes

In December 2012, Chicago Bears head coach Lovie Smith was fired.

Student research idea in Spring 2013: *"I want to prove that Lovie Smith should not have been fired."*

We settled on the idea of trying to separate winning and losing coaches with a multidimensional hyperplane.

Similar ideas have been used in applications ranging from the machine diagnosis of cancer to deciding the authorship of the disputed Federalist Papers.

A plane in n-dimensional space (i.e a hyperplane) is the set of all points $(x_1, x_2, \ldots, x_n)$ satisfying an equation of the form

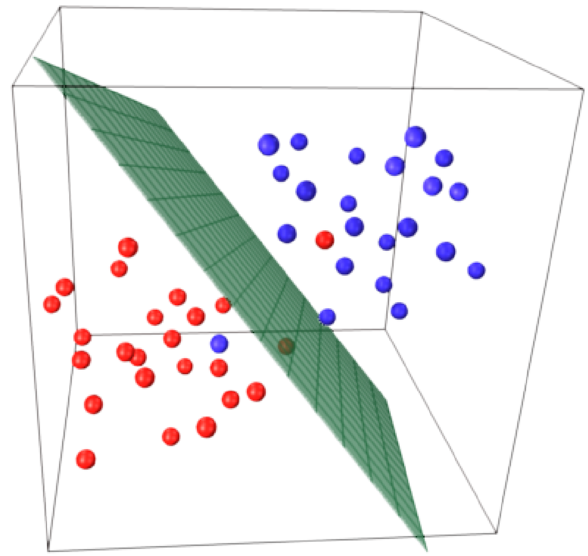
$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b.$$

Two sets of points in n-dimensional space, A and B, are called *linearly separable* if there exists a hyperplane that separates the points, that is,

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n > b \text{ for points in } A$$

and

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n < b \text{ for points in } B.$$



- For separable data, a separating hyperplane is found by solving a certain linear programming problem.

- If data are not linearly separable, one can find a "best" separating hyperplane, which almost separates the data.

Image source: http://sciencepole.com/hyperplane

Lovie Smith was compared only to NFL head coaches whose teams have made it to the playoffs since 2000. There were 52 such coaches.

Excluding Lovie Smith, the 51 remaining coaches were separated into two groups:

Group 1 (Winning Coaches): Coaches were labeled as winning coaches if their team won its conference at some point since 2000. 24 coaches are in this group.

Group 2 (Losing Coaches): Coaches were labeled as losing coaches if their team never won its conference title since 2000. 27 coaches are in this group.

Seven features of coaches were identified, and quantitative data measuring these features were determined for all 52 coaches.

By solving a linear programming problem involving a 51-by-59 matrix, a best separating hyperplane was found.

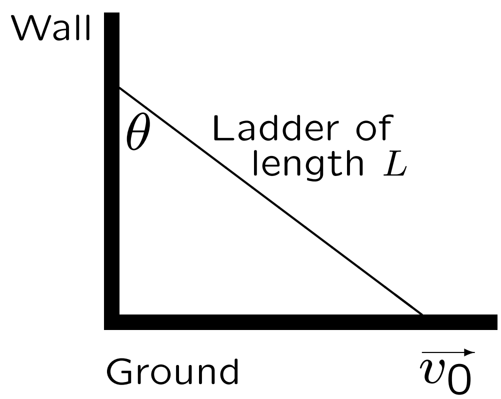
Our hyperplane failed to classify two coaches.

You can think of the 51 coaches as “training data.” They provided training on how to separate coaches.

When Lovie Smith’s features were input, he ended up on the **losing side** of the hyperplane.

Project 2: Falling Ladders and Floating Bobbers

The falling ladder problem is a calculus exercise with a paradoxical twist.



The related-rates solution gives an infinitely fast falling ladder.

Several papers have been written on the falling ladder paradox.

My students noticed that no actual experiments had been reported. So we attempted a Mythbusters-type investigation.

What we discovered was...

- Much remained unsaid about the falling ladder problem.

- The actual experiments were more difficult than we expected.

- There are related-rates “paradoxes” that have attracted no attention.

For example, the floating bobber problem...

- The boat/bobber problem is a famous related-rates problem similar to the falling ladder problem.

- The floating bobber must be eventually be treated as a shrinking pendulum.

- We think the boat in Larson's problem is too close for related rates to apply.

Project 3: Bobo Numbers

In a Classroom Capsule in the CMJ, E. Ray Bobo investigated the sequence $\{a(n)\}_{n=2}^{\infty}$, where $a(n)$ is the least positive integer such that

$$\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{a(n)}>1.$$

Bobo showed that $a(n)$ can only take one of three values: $\lfloor ne\rfloor-2$, $\lfloor ne\rfloor-1$, or $\lfloor ne\rfloor$.

However, Bobo's numerical experiments through $n=2215$ gave no examples where $a(n)=\lfloor ne\rfloor-2$.

Bobo also took a special interest in the set of n's for which $a(n) = \lfloor ne \rfloor$:

$$\begin{aligned} \mathcal{B} &= \{n : a(n) = \lfloor ne \rfloor\} \\ &= \{4, 11, 18, 25, 32, 36, 43, 50, 57, 64, 71, \\ &\quad 75, 82, 89, 96, 103, 114, 121, 128, 135, \\ &\quad 142, 146, 153, 160, 167, 174, 185, 192, \ldots\}. \end{aligned}$$

We call the numbers in this set the *Bobo numbers*.

As a side note, Bobo may be better known for another contribution to the literature:

"You are utterly incorrect about the game show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively towards the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind?"

— E. Ray Bobo to Marilyn vos Savant regarding the Game Show Problem

Bobo posed some questions and my student accepted the challenge.

A: Does the pattern in the gaps of $\mathcal{B}$ persist, or does chaos eventually take over?

B: Is $a(n) = \lfloor ne \rfloor - 2$ impossible?

Our research answered Bobo's questions, won an award, got published, and gave rise to new questions.

For example, we found that if n is an integer greater than 2 satisfying

$$\{ne\} \geq \frac{1}{2}(e-1),$$

then n must be a Bobo number.

The converse is not true—there are exceptional Bobo numbers.

Project 4: Exceptional Bobo Numbers

The exceptional Bobo numbers are rare.

In a list of the first 845 million Bobo numbers, only four are exceptional:

36,   9045,   5195512,   5311399545.

In a spring 2016 project, two of my students took on the task of finding and describing the exceptional Bobo numbers.

Our research involved countless hours of computer searches and lots of time looking for patterns.

Our search for patterns eventually led us to Farey fractions and then to continued fractions.

- There are infinitely many exceptional Bobo numbers, and they can all be obtained from the denominators of the odd-indexed convergents of the continued fraction for $(e-1)/2$.

- There are two types of exceptional Bobo numbers.

  – Type-1 EBNs are those obtained directly from the continued fraction convergents. Their computation requires only integer arithmetic.

  – Type-2 EBNs are computed from Type-1 EBNs. They must be confirmed as exceptional using high-precision floating-point arithmetic.

The EBNs...

36
9045
5195512
5311399545
8488859795196
25466579385587
1954296585112062 1
58628897553361862
612507720048708415 20
1837523160146125245 59
250769086731739376780337
752307260195218130341010
12995157350217026255449760 20
38985472050651078766349280 59
83144418829771650417283168946 61
2494332564893149512518495068 3982
6437121879677320767217157485 5623976
19311365639031962301651472456 6871927
59312212282608022053471337164 4428080425
17793663684782406616041401149 33284241274
...

More details about the projects can be found in the handout. Even more details can be found by asking me—please feel free to do so!

Thanks for attending.

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Slides and handouts are available at
http://pubs.stevekifowit.com