Exponent Laws

For any positive integer n,

$$a^n = \underbrace{a \cdot a \cdots a}_{n \text{ times}} \tag{1}$$

For any number $a \neq 0$,

$$a^0 = 1 \tag{2}$$

For any number $n \neq 0$,

$$0^n = 0 \tag{3}$$

For any numbers n and $a \neq 0$,

$$a^{-n} = \frac{1}{a^n} \tag{4}$$

For any numbers n and m and $a \neq 0$,

$$a^n a^m = a^{n+m} \tag{5}$$

$$\frac{a^n}{a^m} = a^{n-m} \tag{6}$$

$$(a^n)^m = a^{nm} \tag{7}$$

For any numbers n and $a, b \neq 0$,

$$(ab)^n = a^n b^n \tag{8}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \tag{9}$$

For integers n and m and any number a,

$$a^{n/m} = \sqrt[m]{a^n} = (\sqrt[m]{a})^n \tag{10}$$

provided these are defined.

Factoring

To factor $x^2 + Bx + C$ over the integers:

1. Identify coefficients B and C.
2. Find two integers P and Q such that

   $$PQ = C \quad \text{and} \quad P + Q = B$$

3. The factorization is

   $$x^2 + Bx + C = (x + P)(x + Q)$$

To factor $Ax^2 + Bx + C$ over the integers:

1. Identify coefficients A, B, and C.
2. Find two integers P and Q such that

   $$PQ = AC \quad \text{and} \quad P + Q = B$$

3. Rewrite the original trinomial in the form

   $$Ax^2 + Px + Qx + C$$

4. The polynomial now factors by grouping.

Perfect Squares

$$A^2 + 2AB + B^2 = (A + B)^2$$

Differences of Squares

$$A^2 - B^2 = (A + B)(A - B)$$

Sums and Differences of Cubes

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$