Power Reducing Formulae

1. For positive integers n...

   (a) $$\sin^2 x = \frac{1}{2} \cdot (1 - \cos 2x)$$

   (b) $$\sin^3 x = \frac{1}{4} \cdot (3 \sin x - \sin 3x)$$

   (c) $$\sin^4 x = \frac{1}{8} \cdot (3 - 4\cos 2x + \cos 4x)$$

   (d) $$\sin^5 x = \frac{1}{16} \cdot (\sin 5x - 5 \sin 3x + 10 \sin x)$$

   (e) $$\sin^6 x = \frac{1}{32} \cdot (10 - 15 \cos 2x + 6 \cos 4x - \cos 6x)$$

   (f) $$\sin^{2n-1} x = \frac{(-1)^{n-1}}{2^{2n-2}} \cdot \left[\sin(2n-1)x - \binom{2n-1}{1} \sin(2n-3)x + \cdots + (-1)^{n-1} \binom{2n-1}{n-1} \sin x \right]$$

   (g) $$\sin^{2n} x = \frac{1}{2^{2n}} \binom{2n}{n} + \frac{(-1)^n}{2^{2n-1}} \cdot \left[\cos 2nx - \binom{2n}{1} \cos(2n-2)x + \cdots + (-1)^{n-1} \binom{2n}{n-1} \cos 2x \right]$$

2. For positive integers n...

   (a) $$\cos^2 x = \frac{1}{2} \cdot (1 + \cos 2x)$$

   (b) $$\cos^3 x = \frac{1}{4} \cdot (3 \cos x + \cos 3x)$$

   (c) $$\cos^4 x = \frac{1}{8} \cdot (3 + 4 \cos 2x + \cos 4x)$$

   (d) $$\cos^5 x = \frac{1}{16} \cdot (\cos 5x + 5 \cos 3x + 10 \cos x)$$

   (e) $$\cos^6 x = \frac{1}{32} \cdot (\cos 6x + 6 \cos 4x + 15 \cos 2x + 10)$$

   (f) $$\cos^{2n-1} x = \frac{1}{2^{2n-2}} \cdot \left[\cos(2n-1)x + \binom{2n-1}{1} \cos(2n-3)x + \cdots + \binom{2n-1}{n-1} \cos x \right]$$

   (g) $$\cos^{2n} x = \frac{1}{2^{2n}} \binom{2n}{n} + \frac{1}{2^{2n-1}} \cdot \left[\cos 2nx + \binom{2n}{1} \cos(2n-2)x + \cdots + \binom{2n}{n-1} \cos 2x \right]$$

In each case above,

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$