Definition of Riemann Sum

Let f be defined on the closed interval $[a, b]$, and let Δ be a partition of $[a, b]$ given by

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

where Δx_i is the length of the ith subinterval. If c_i is any point in the ith subinterval, then the sum

$$\sum_{i=1}^{n} f(c_i)\Delta x_i, \quad x_{i-1} \le c_i \le x_i$$

is called a Riemann sum of f for the partition Δ.

Definition of Definite Integral

If f is defined on the closed interval $[a, b]$ and the limit

$$\lim_{||\Delta|| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

exists, then f is Riemann integrable on $[a, b]$ and the limit is denoted by

$$\lim_{||\Delta|| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)dx.$$

The limit is called the definite integral of f from a to b. The numbers a and b are called the lower and upper limits of integration, respectively.

Continuity Implies Integrability

If a function f is continuous on the closed interval $[a, b]$, then f is Riemann integrable on $[a, b]$.

The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a).$$

The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then, for every x in I,

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x).$$