Summary of Convergence Tests

Test	Series	Converges	Diverges	Comments
Telescoping Series	$\sum_{n=1}^{\infty}(a_n - a_{n+1})$	$\lim_{n\to\infty} a_{n+1} = L \neq \infty$	$\lim_{n\to\infty} a_{n+1}$ DNE	Often requires partial fraction decomposition. Converges to $a_1 - \lim_{n\to\infty} a_{n+1}$
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	Converges to $\frac{a}{1-r}$
nth-Term Test	$\sum_{n=1}^{\infty} a_n$	Cannot be used to show convergence.	$\lim_{n\to\infty} a_n \neq 0$	Cannot be used to show convergence.
Integral Test (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n, \quad a_n = f(n)$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	Remainder: $0 < R_n < \int_N^{\infty} f(x)\,dx$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	Often used along with a comparison.
Logarithmic p-Series	$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$	$p > 1$	$p \leq 1$	
Direct Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} a_n/b_n = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} a_n/b_n = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$	Cannot be used to show divergence.	Remainder: $\lvert R_N\rvert \leq a_{N+1}$
Ratio Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	Test is inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$
nth-Root Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$	Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$