Integrals of the form $\int \tan^n x \sec^m x \, dx$

1. If the power of $\tan x$ is odd and positive, save a secant-tangent factor and convert the remaining tangents to secants. Then let $u = \sec x$.

$$\begin{aligned}
\int \tan^{\overbrace{2k+1}^{\text{Odd}}} x \sec^m x \, dx &= \int \overbrace{(\tan^2 x)^k}^{\text{Convert}} \sec^{m-1} x \overbrace{\sec x \tan x \, dx}^{\text{Save for } du} \\
&= \int (\sec^2 x - 1)^k \sec^{m-1} x \sec x \tan x \, dx \\
&= \int (u^2 - 1)^k u^{m-1} \, du
\end{aligned}$$

2. If the power of $\sec x$ is even and positive, save one $\sec^2 x$ factor and convert the remaining secants to tangents. Then let $u = \tan x$.

$$\begin{aligned}
\int \tan^n x \sec^{\overbrace{2k+2}^{\text{Even}}} x \, dx &= \int \tan^n x \overbrace{(\sec^2 x)^k}^{\text{Convert}} \overbrace{\sec^2 x \, dx}^{\text{Save for } du} \\
&= \int \tan^n x \, (1 + \tan^2 x)^k \sec^2 x \, dx \\
&= \int u^n (1 + u^2)^k \, du
\end{aligned}$$

3. If there are no secants and the power of $\tan x$ is even and positive, convert a $\tan^2 x$ factor to a $\sec^2 x$ factor. Then expand, use #2, and repeat if necessary.

$$\begin{aligned}
\int \tan^{\overbrace{2k+2}^{\text{Even}}} x \, dx &= \int \tan^{2k} x \overbrace{\tan^2 x}^{\text{Convert}} \, dx \\
&= \int \tan^{2k} x \, (\sec^2 x - 1) \, dx \\
&= \int \tan^{2k} x \sec^2 x \, dx - \int \tan^{2k} x \, dx
\end{aligned}$$

4. If the integral has the form $\int \sec^m x \, dx$ where m is odd and positive, use integration-by-parts by setting $f(x) = \sec^{m-2} x$ and $g'(x) = \sec^2 x$.

5. If all else fails, try converting to sines and cosines.