

Trigonometric Substitution

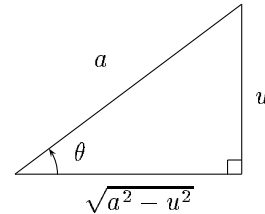
(Assuming $a > 0$)

- (1) For integrals involving $\sqrt{a^2 - u^2}$, use the substitution

$$u = a \sin \theta \quad du = a \cos \theta d\theta$$

where $-\pi/2 \leq \theta \leq \pi/2$. It follows that

$$\sqrt{a^2 - u^2} = a \cos \theta.$$

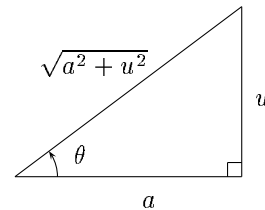


- (2) For integrals involving $\sqrt{a^2 + u^2}$, use the substitution

$$u = a \tan \theta \quad du = a \sec^2 \theta d\theta$$

where $-\pi/2 < \theta < \pi/2$. It follows that

$$\sqrt{a^2 + u^2} = a \sec \theta.$$



- (3) For integrals involving $\sqrt{u^2 - a^2}$, use the substitution

$$u = a \sec \theta \quad du = a \sec \theta \tan \theta d\theta$$

where $0 \leq \theta < \pi/2$ or $\pi/2 < \theta \leq \pi$. It follows that

$$\sqrt{u^2 - a^2} = \pm a \tan \theta.$$

Use the positive square root if $u > a$ and use the negative square root if $u < -a$.

