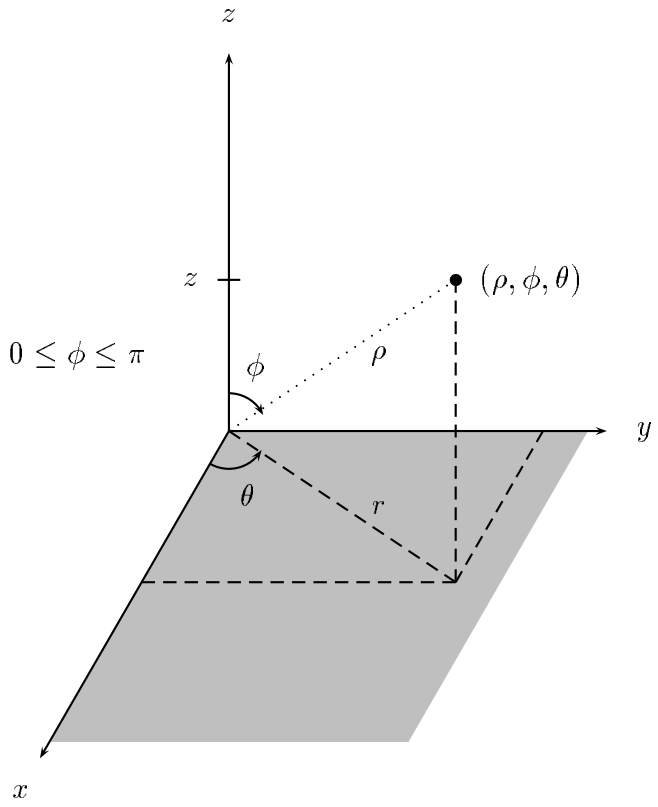
Spherical Coordinates



$$x = \rho \sin\phi \cos\theta \qquad \rho^2 = x^2 + y^2 + z^2$$
$$y = \rho \sin\phi \sin\theta \qquad \cos\phi = z/\rho$$
$$z = \rho \cos\phi \qquad \tan\theta = y/x$$

$$\hat{i} = \sin\phi\,\cos\theta\,\hat{\rho} + \cos\phi\,\cos\theta\,\hat{\phi} - \sin\theta\,\hat{\theta} \qquad \hat{\rho} = \sin\phi\,\cos\theta\,\hat{i} + \sin\phi\,\sin\theta\,\hat{j} + \cos\phi\,\hat{k}$$
$$\hat{j} = \sin\phi\,\sin\theta\,\hat{\rho} + \cos\phi\,\sin\theta\,\hat{\phi} + \cos\theta\,\hat{\theta} \qquad \hat{\phi} = \cos\phi\,\cos\theta\,\hat{i} + \cos\phi\,\sin\theta\,\hat{j} - \sin\phi\,\hat{k}$$
$$\hat{k} = \cos\phi\,\hat{\rho} - \sin\phi\,\hat{\phi} \qquad \hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$d\vec{s} = d\rho\,\hat{\rho} + \rho\,d\phi\,\hat{\phi} + \rho\sin\phi\,d\theta\,\hat{\theta}; \quad dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$$

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{1}{\rho\sin\phi}\frac{\partial f}{\partial \theta}\hat{\theta}$$

$$\nabla \cdot \vec{v} = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}(\rho^2 v_\rho) + \frac{1}{\rho\sin\phi}\frac{\partial}{\partial \phi}(v_\phi \sin\phi) + \frac{1}{\rho\sin\phi}\frac{\partial v_\theta}{\partial \theta}$$

$$\nabla \times \vec{v} = \frac{1}{\rho\sin\phi}\left[\frac{\partial}{\partial\phi}(v_\theta \sin\phi) - \frac{\partial v_\phi}{\partial\theta}\right]\hat{\rho} + \frac{1}{\rho}\left[\frac{1}{\sin\phi}\frac{\partial v_\rho}{\partial\theta} - \frac{\partial}{\partial\rho}\rho(v_\theta)\right]\hat{\phi} + \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho v_\phi) + \frac{\partial v_\rho}{\partial\phi}\right]\hat{\theta}$$

$$\nabla^2 f = \frac{1}{\rho^2}\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial f}{\partial\phi}\right) + \frac{1}{\rho^2\sin^2\phi}\frac{\partial^2 f}{\partial\theta^2}$$