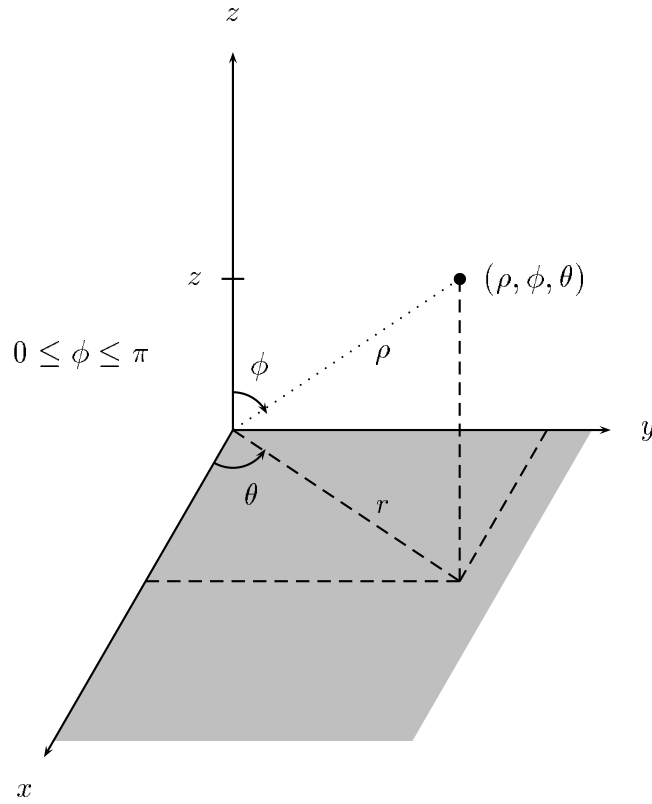


Spherical Coordinates



$$\begin{aligned}x &= \rho \sin \phi \cos \theta \\y &= \rho \sin \phi \sin \theta \\z &= \rho \cos \phi\end{aligned}$$

$$\begin{aligned}\rho^2 &= x^2 + y^2 + z^2 \\ \cos \phi &= z/\rho \\ \tan \theta &= y/x\end{aligned}$$

$$\begin{aligned}\hat{i} &= \sin \phi \cos \theta \hat{\rho} + \cos \phi \cos \theta \hat{\phi} - \sin \theta \hat{\theta} \\ \hat{j} &= \sin \phi \sin \theta \hat{\rho} + \cos \phi \sin \theta \hat{\phi} + \cos \theta \hat{\theta} \\ \hat{k} &= \cos \phi \hat{\rho} - \sin \phi \hat{\phi}\end{aligned}$$

$$\begin{aligned}\hat{\rho} &= \sin \phi \cos \theta \hat{i} + \sin \phi \sin \theta \hat{j} + \cos \phi \hat{k} \\ \hat{\phi} &= \cos \phi \cos \theta \hat{i} + \cos \phi \sin \theta \hat{j} - \sin \phi \hat{k} \\ \hat{\theta} &= -\sin \theta \hat{i} + \cos \theta \hat{j}\end{aligned}$$

$$d\vec{s} = d\rho \hat{\rho} + \rho d\phi \hat{\phi} + \rho \sin \phi d\theta \hat{\theta}; \quad dV = \rho^2 \sin \phi d\rho d\phi d\theta$$

$$\nabla f = \frac{\partial f}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{1}{\rho \sin \phi} \frac{\partial f}{\partial \theta} \hat{\theta}$$

$$\nabla \cdot \vec{v} = \frac{1}{\rho^2} \frac{\partial}{\partial \rho} (\rho^2 v_\rho) + \frac{1}{\rho \sin \phi} \frac{\partial}{\partial \phi} (v_\phi \sin \phi) + \frac{1}{\rho \sin \phi} \frac{\partial v_\theta}{\partial \theta}$$

$$\nabla \times \vec{v} = \frac{1}{\rho \sin \phi} \left[\frac{\partial}{\partial \phi} (v_\theta \sin \phi) - \frac{\partial v_\phi}{\partial \theta} \right] \hat{\rho} + \frac{1}{\rho} \left[\frac{1}{\sin \phi} \frac{\partial v_\rho}{\partial \theta} - \frac{\partial}{\partial \rho} \rho (v_\theta) \right] \hat{\phi} + \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho v_\phi) + \frac{\partial v_\rho}{\partial \phi} \right] \hat{\theta}$$

$$\nabla^2 f = \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left(\rho^2 \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2 \sin \phi} \frac{\partial}{\partial \phi} \left(\sin \phi \frac{\partial f}{\partial \phi} \right) + \frac{1}{\rho^2 \sin^2 \phi} \frac{\partial^2 f}{\partial \theta^2}$$