Basis Vectors

$$\mathbf{i} = \langle 1, 0, 0 \rangle$$

$$\mathbf{j} = \langle 0, 1, 0 \rangle$$

$$\mathbf{k} = \langle 0, 0, 1 \rangle$$

$$\mathbf{u} = \langle u_1, u_2, u_3 \rangle = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$$

Magnitude

$$\|\mathbf{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

Dot Product

$$\mathbf{u} \cdot \mathbf{w} = u_1 w_1 + u_2 w_2 + u_3 w_3$$

$$\mathbf{u} \cdot \mathbf{w} = \|\mathbf{u}\| \, \|\mathbf{w}\| \cos\theta$$

Projection

$$\operatorname{proj}_{\mathbf{w}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{w}}{\mathbf{w} \cdot \mathbf{w}} \right) \mathbf{w}$$

Cross Product

$$\mathbf{u} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\|\mathbf{u} \times \mathbf{w}\| = \|\mathbf{u}\| \, \|\mathbf{w}\| \sin\theta$$

Position, Velocity, Acceleration

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

$$\mathbf{v}(t) = \mathbf{r}'(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j} + z'(t)\mathbf{k}$$

$$\mathbf{a}(t) = \mathbf{r}''(t) = x''(t)\mathbf{i} + y''(t)\mathbf{j} + z''(t)\mathbf{k}$$

Arc Length

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, dt$$

$$L = \int_a^b \|\mathbf{v}(t)\| \, dt$$

$$s(t) = \int_{t_0}^t \|\mathbf{v}(\tau)\| \, d\tau, \qquad \frac{ds}{dt} = \|\mathbf{v}(t)\|$$

Unit Tangent Vector

$$\mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

Curvature

$$\kappa = \left\| \frac{d\mathbf{T}}{ds} \right\| = \frac{1}{\|\mathbf{v}\|} \left\| \frac{d\mathbf{T}}{dt} \right\| = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|^3}$$

$$y = f(x) \quad \Rightarrow \quad \kappa = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}$$

Principal Unit Normal Vector

$$\mathbf{N} = \frac{1}{\kappa} \frac{d\mathbf{T}}{ds} = \frac{d\mathbf{T}/dt}{\|d\mathbf{T}/dt\|}$$

Osculating Circle

$$\text{radius: } \rho = \frac{1}{\kappa(t_0)}$$

$$\text{center: } \mathbf{C} = \mathbf{r}(t_0) + \frac{1}{\kappa(t_0)} \mathbf{N}(t_0)$$

Unit Binormal Vector

$$\mathbf{B} = \mathbf{T} \times \mathbf{N}$$

Torsion

$$\tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N} = \frac{\begin{vmatrix} x'(t) & y'(t) & z'(t) \\ x''(t) & y''(t) & z''(t) \\ x'''(t) & y'''(t) & z'''(t) \end{vmatrix}}{\|\mathbf{v} \times \mathbf{a}\|^2}$$

Acceleration

$$\mathbf{a} = a_T \, \mathbf{T} + a_N \, \mathbf{N}$$

$$a_T = \frac{d}{dt}\|\mathbf{v}\| = \frac{\mathbf{v} \cdot \mathbf{a}}{\|\mathbf{v}\|}$$

$$a_N = \kappa \|\mathbf{v}\|^2 = \sqrt{\|\mathbf{a}\|^2 - a_T^2} = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|}$$

Projectile Motion

$$\mathbf{r}(t) = ((v_0 \cos\theta)t + x_0)\,\mathbf{i} + \left(-\frac{1}{2}gt^2 + (v_0 \sin\theta)t + y_0\right)\mathbf{j}$$

Gradient Vector

$$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

Directional Derivative

$$D_{\mathbf{u}} f = \frac{1}{\|\mathbf{u}\|} \left(\nabla f \cdot \mathbf{u} \right)$$