# Euler's Method

Suppose we wish to solve the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0.$$

*Euler's method* is a numerical algorithm for estimating points on the graph of the solution curve. The approach generates approximate values by taking steps along local linearizations, starting at the known point $(x_0, y_0)$.

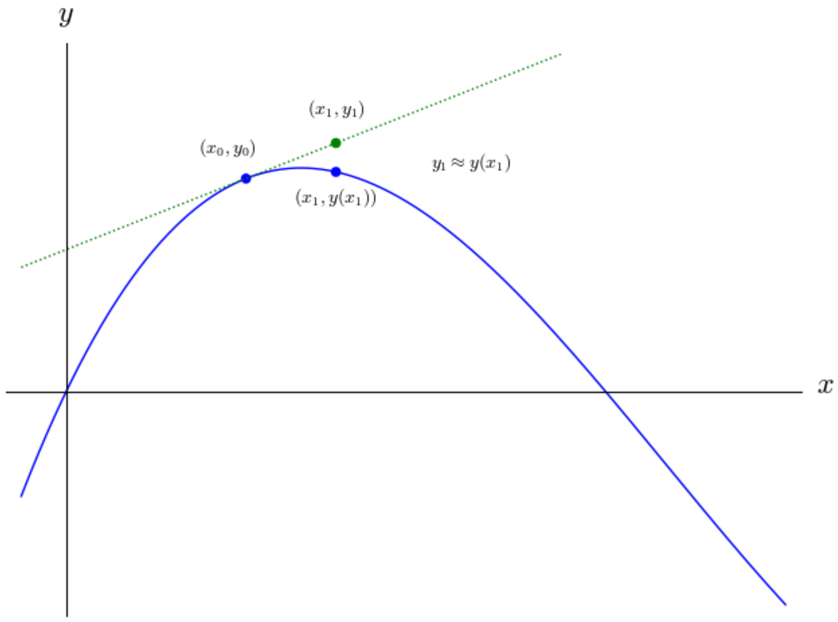
We derive Euler's method as follows:

1. Using the equation and initial condition, find the linearization, $L(x)$, of $y(x)$ at $(x_0, y_0)$:

$$L(x) = y(x_0) + f(x_0, y_0)(x - x_0) = y_0 + f(x_0, y_0)(x - x_0).$$

2. Let $x_1 = x_0 + h$, where $h$ is the size of the step you would like to take.
3. Use the linearization (see figure below) to approximate $y(x_1)$ and name your approximate value $y_1$:

$$y(x_1) \approx y_1 = L(x_1) = y_0 + f(x_0, y_0)(x_1 - x_0) = y_0 + h\, f(x_0, y_0).$$




4. Now repeat the entire process using $(x_1, y_1)$ as the starting point and thereby generating $(x_2, y_2)$, where $x_2 = x_1 + h$ and $y_2 \approx y(x_2)$.
5. Continue to repeat. Euler's method is summarized by the recursive formulas

$$y_{n+1} = y_n + h\, f(x_n, y_n),$$
$$x_{n+1} = x_n + h; \quad n = 0, 1, 2, 3, \ldots$$

**Roughly speaking, the error in the approximation at each step is proportional to the step size $h$. Smaller step sizes tend to produce smaller errors. However the errors may accumulate as you take more steps.**