

A *direction field* or *slope field* for a differential equation is a plot of short line segments drawn at various points in the  $xy$ -plane showing the slopes of the solution curves at those points.

## Existence Theorem (Cauchy-Peano)

Suppose we are given the IVP

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0.$$

If  $f$  is continuous on the rectangle

$$R = \{(x, y) : a \leq x \leq b, c \leq y \leq d\}$$

containing  $(x_0, y_0)$ , then the IVP has a solution on the interval  $(x_0 - h, x_0 + h)$  for some positive number  $h$ .

Unfortunately, this result only tells us that the IVP has a solution in a neighborhood of  $x_0$ . We have no idea how big that neighborhood actually is.

## Existence/Uniqueness

Suppose we are given the IVP

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0.$$

If  $f$  and  $\partial f/\partial y$  are continuous on the rectangle

$$R = \{(x, y) : a \leq x \leq b, c \leq y \leq d\}$$

containing  $(x_0, y_0)$ , then the IVP has a unique solution on the interval  $(x_0 - h, x_0 + h)$  for some positive number  $h$ .

As above, we generally cannot say much about the number  $h$ . If the DE has a special form, we may be able to say more.