Suppose that p_1, p_2, . . . , p_n, and f are continuous on an open interval I containing a. Then the linear ODE

$$y^{(n)} + p_1(x)y^{(n-1)} + \cdots + p_n(x)y = f(x)$$

has a unique solution on the entire interval I for any choice of initial values for

$$y(a),\ y'(a),\ y''(a),\ \ldots,\ y^{(n-1)}(a).$$

On the open interval I, suppose y_1, y_2, . . . , y_n are solutions of the homogeneous, linear ODE

$$y^{(n)} + p_1(x)y^{(n-1)} + \cdots + p_n(x)y = 0.$$

Then, for any constants c_1, c_2, . . . , c_n,

$$y(x) = c_1y_1(x) + c_2y_2(x) + \cdots + c_ny_n(x)$$

is also a solution on I.

On the open interval I, suppose that y_p is a particular solution of

$$y^{(n)} + p_1(x)y^{(n-1)} + \cdots + p_n(x)y = f(x)$$

and that y_1, y_2, . . . , y_n are linearly independent solutions of the corresponding homogeneous equation. Then every solution of

$$y^{(n)} + p_1(x)y^{(n-1)} + \cdots + p_n(x)y = f(x)$$

has the form

$$y(x) = y_p(x) + c_1y_1(x) + c_2y_2(x) + \cdots + c_ny_n(x),$$

where c_1, c_2, . . . , c_n are constants.

On the open interval I, suppose that $y_{p_1}(x)$ is a solution of

$$y^{(n)} + p_1(x)y^{(n-1)} + \cdots + p_n(x)y = g_1(x)$$

and $y_{p_2}(x)$ is a solution of

$$y^{(n)} + p_1(x)y^{(n-1)} + \cdots + p_n(x)y = g_2(x).$$

Then $y(x) = Ky_{p_1}(x) + My_{p_2}(x)$ is a solution of

$$y^{(n)}+p_1(x)y^{(n-1)}+\cdots+p_n(x)y = Kg_1(x)+Mg_2(x)$$

on I.

This is called a *superposition principle*.