Theorem: Suppose $f(t)$ is continuous on $[0, \infty)$ and $f'(t)$ is piecewise continuous on $[0, \infty)$, both with exponential order α. Then, for $s > \alpha$,

$$\mathscr{L}\{f'\}(s) = s\,\mathscr{L}\{f\}(s) - f(0).$$

Notice that the theorem can be applied repeatedly by replacing f with f'.

Under the appropriate conditions on f, f', and f'', we have

$$\mathscr{L}\{f''\}(s) = s\,\mathscr{L}\{f'\}(s) - f'(0)$$

$$= s\,[s\,\mathscr{L}\{f\}(s) - f(0)] - f'(0)$$

$$= s^2\,\mathscr{L}\{f\}(s) - s\,f(0) - f'(0).$$

Theorem: Suppose $f(t), f'(t), \ldots f^{(n-1)}(t)$ are continuous on $[0, \infty)$ and $f^{(n)}(t)$ is piecewise continuous on $[0, \infty)$, all with exponential order α. Then, for $s > \alpha$,

$$\mathscr{L}\{f^{(n)}\}(s) = s^n \mathscr{L}\{f\}(s) - s^{n-1} f(0)$$

$$-s^{n-2} f'(0) - \cdots - f^{(n-1)}(0).$$