Periodic functions

Suppose $f(t)$ is defined for all t. If there is a number p such that

$$f(t + p) = f(t)$$

for all t, then f is *periodic* with *period* p.

Fourier Series

Let $f(t)$ be a piecewise continuous function of period 2π that is defined for all t. Then the *Fourier series* of $f(t)$ is the function

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$$

where the *Fourier coefficients* are given by

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt \, dt, \qquad n = 0, 1, 2, 3, \ldots$$

and

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt \, dt, \qquad n = 1, 2, 3, \ldots.$$

To denote the Fourier series, we will write

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt).$$