General Fourier Series

Let $f(t)$ be a piecewise continuous function of period $2L$ that is defined for all t. Then the *Fourier series* of $f(t)$ is the function

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L} \right)$$

where the *Fourier coefficients* are given by

$$a_n = \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi t}{L} \, dt, \quad n = 0, 1, 2, 3, \ldots$$

and

$$b_n = \frac{1}{L} \int_{-L}^{L} f(t) \sin \frac{n\pi t}{L} \, dt, \qquad n = 1, 2, 3, \ldots.$$

Convergence of Fourier Series

Theorem: Suppose that the periodic function f is piecewise smooth. Then its Fourier series converges

1. to the value $f(t)$ at each point where f is continuous, and

2. to the value
$$\frac{f(t+) + f(t-)}{2}$$
at each point where f is discontinuous.