

$$\underline{E_x} \quad t x'' + (3t-1)x' + 3x = 0, \quad x(0) = 0$$

$$\mathcal{L}\{t x''\} + 3\mathcal{L}\{t x'\} - \mathcal{L}\{x'\} + 3\mathcal{L}\{x\} = 0$$

$$-\frac{d}{ds} \left(s^2 X(s) - s x(0) - x'(0) \right) - 3 \frac{d}{ds} \left(s X(s) - x(0) \right) + 3X(s) = 0$$

$$-3 \frac{d}{ds} \left(s X(s) - x(0) \right) - \left(s X(s) - x(0) \right) + 3X(s) = 0$$

$$-s^2 X'(s) - 2s X(s) - 3s X'(s) - 3X(s) - s X(s) + 3X(s) = 0$$

$$(-s^2 - 3s) X'(s) - 3s X(s) = 0$$

$$\frac{dX}{ds} = \frac{-3s}{s^2 + 3s} X$$

$$\frac{dX}{X} = \frac{-3}{s+3} ds$$

$$\ln |X| = -3 \ln |s+3| + C$$

$$\ln |X| = \ln \frac{1}{|s+3|^3} + C$$

$$|X| = e^C \left(\frac{1}{|s+3|^3} \right)$$

$$X(s) = \frac{C}{(s+3)^3}; \quad s > -3, \quad C > 0$$

$$x(t) = \frac{C}{6} t^2 e^{-3t}, \quad C > 0$$