Ex $y'' + y = 0$

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$y''(x) = \underbrace{\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}}_{n+2 \to n}$$

$$y'' + y = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\Rightarrow (n+2)(n+1) a_{n+2} + a_n = 0$$

$$a_{n+2} = -\frac{1}{(n+2)(n+1)} a_n$$

EVEN INDICIES...

$$a_2 = -\frac{1}{2} a_0$$

$$a_4 = -\frac{1}{4 \cdot 3} \cdot \left(-\frac{1}{2} a_0\right)$$

$$a_6 = -\frac{1}{6 \cdot 5} \cdot \left(\frac{1}{4 \cdot 3 \cdot 2} a_0\right)$$

$\vdots$

$$a_{2n} = \frac{(-1)^n}{(2n)!} a_0 \; ; \; n = 1, 2, 3, \ldots$$

ODD INDICIES...

$$a_{2n+1} = \frac{(-1)^n}{(2n+1)!} a_1 \; ; \; n = 1, 2, 3, \ldots$$

LET a_0 & a_1 BE OUR TWO CONSTANTS OF INTEGRATION.

$$y(x) = \sum_{n=0}^{\infty} a_0 \frac{(-1)^n}{(2n)!} x^{2n} + \sum_{n=0}^{\infty} a_1 \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\Rightarrow y(x) = a_0 \cos X + a_1 \sin X$$