Ex Solve $y'' + 2ty' - 4y = 1$, $y(0) = y'(0) = 0$

$$\mathcal{L}\{y''\} + 2\mathcal{L}\{ty'\} - 4\mathcal{L}\{y\} = \frac{1}{s}$$

$$\left[s^2 Y(s) - sy(0) - y'(0)\right] + \underbrace{2(-1)\frac{d}{ds}\mathcal{L}\{y'\}}_{-2\frac{d}{ds}\left(sY(s) - y(0)\right)} - 4Y(s) = \frac{1}{s}$$

$$s^2 Y(s) - 2\frac{d}{ds}\left(sY(s)\right) - 4Y(s) = \frac{1}{s}$$

$$s^2 Y(s) - 2Y(s) - 2sY'(s) - 4Y(s) = \frac{1}{s}$$

$$-2sY'(s) + (s^2 - 6)Y(s) = \frac{1}{s}$$

$$Y'(s) - \frac{s^2-6}{2s}Y(s) = -\frac{1}{2s^2} \quad \star$$

1st order linear. Solution on next page. (Good review!)

$$Y(s) = \frac{1}{s^3} + C\frac{e^{s^2/4}}{s^3}, \quad C \text{ is constant.}$$

Recall that transforms must approach zero as $s \to \infty$.

Therefore, we must have $C = 0$.

$$Y(s) = \frac{1}{s^3} \Rightarrow \boxed{y(t) = \frac{1}{2}t^2}$$

☆

$$Y' - \frac{s^2-6}{2s} Y = -\frac{1}{2s^2}$$

$$\mu(s) = e^{\int -\frac{s^2-6}{2s} ds} = e^{\int\left(-\frac{s}{2}+\frac{3}{s}\right) ds}$$

$$= e^{-\frac{s^2}{4} + 3\ln s}$$

$$= s^3 e^{-s^2/4}$$

$$\mu(s) Y(s) = \int \mu(s) \left(-\frac{1}{2s^2}\right) ds$$

$$= \int -\frac{s}{2} e^{-s^2/4} ds \qquad u = -\frac{s^2}{4}$$

$$du = -\frac{s}{2} ds$$

$$= \int e^u du = e^u + C$$

$$= e^{-s^2/4} + C$$

$$Y(s) = \frac{1}{\mu(s)} \left[e^{-s^2/4} + C \right]$$

$$Y(s) = \frac{1}{s^3} + \frac{C}{s^3 e^{-s^2/4}}$$

$$Y(s) = \frac{1}{s^3} + \frac{C e^{s^2/4}}{s^3}$$