

- Completely factor the polynomial $f(x) = x^4 - 8x^3 + 200x - 625$.

THERE IS NOTHING THAT IS OBVIOUS OR EASY, SO

LET'S START BY THINKING ABOUT RATIONAL ZEROS.

POSSIBLE RATIONAL ZEROS ARE

$$\frac{\pm \text{FACTOR OF } 625}{\text{FACTOR OF } 1} = \pm \{5, 25, 125, 625\}$$

By using a calculator, we find that $f(5) = 0$
AND $f(-5) = 0$.

So $(x-5)(x+5)$ ARE FACTORS,
 $x^2 - 25$ DIVIDE OUT $x^2 - 25$

$$\begin{array}{r} x^2 - 8x + 25 \\ x^2 - 25 \overline{) x^4 - 8x^3 + 0x^2 + 200x - 625} \\ - (x^4 \quad - 25x^2) \\ \hline -8x^3 + 25x^2 + 200x - 625 \\ - (-8x^3 \quad + 200x) \\ \hline 25x^2 - 625 \\ - (25x^2 - 625) \\ \hline 0 \end{array}$$

Now we only have to factor $x^2 - 8x + 25$.

Q.F...

$$\begin{aligned} x &= \frac{8 \pm \sqrt{64 - 4(1)(25)}}{2} \\ &= \frac{8 \pm \sqrt{-36}}{2} = \frac{8 \pm 6i}{2} \\ &= 4 \pm 3i \end{aligned}$$

$$f(x) = (x-5)(x+5)(x-(4+3i))(x-(4-3i))$$