Lecture 2: Review: Functions, Change, and Graphing

Objectives:

(2.1) Evaluate functions.

(2.2) Compute and interpret Δx and Δy.

(2.3) Sketch basic graphs by hand.

Functions

A function from a set A into a set B is a rule or correspondence that assigns to **each** element of A a **single** element of B. The set A is called the domain of the function—the domain is the set of all possible inputs. The set of all outputs is called the range of the function. The range is a subset of B, not necessarily all of B. A variable used to name a domain input is called an independent variable, while a variable used to name a range output is called a dependent variable.

Functions can be defined in many ways: in words; with diagrams, tables, or graphs; with equations; etc. Most of the functions we will study will be defined algebraically—the correspondence defining the function will be described by an equation.

Notice that the domain of a function is a defining characteristic of the function. The domain must be given! The same rule applied on different domains defines different functions. We will adopt the following convention:

If the domain of a function is not explicitly given, we will assume the domain is the set of all real numbers that make sense in the context of the function's definition.

Example 1 Let $f(x) = \dfrac{1}{1-x}$.

1. Evaluate $f(\frac{1}{2})$.
2. Find all x-values for which $f(x) = -5$.
3. Find the domain and range of f.

Solutions omitted.

Example 2 If Fred sells his Whatchies for x dollars apiece, he makes a profit of $p(x) = x^2 + 2x - 4$ dollars for each one he sells.

1. What is the domain of p?

   Since x represents a number of dollars, it only makes sense that $x \geq 0$.

2. Complete the square to find the range of p.

$$x^2 + 2x - 4 = (x+1)^2 - 5 \geq -5$$

3. Simplify and interpret $p(x+1)$.

   $p(x+1)$ is Fred's profit per Watchie after selling them for $x+1$ dollars apiece. This may be of interest to Fred if he is considering raising his price by \$1.

$$p(x+1) = (x+1)^2 + 2(x+1) - 4 = x^2 + 2x + 1 + 2x + 2 - 4 = x^2 + 4x - 1$$

   Notice that $p(x+1) - p(x) = 2x + 3$ is Fred's change in profit per Watchie if he raises his price \$1.

Change

If the value of x changes from $x = x_{old}$ to $x = x_{new}$, the change in x is denoted by Δx:

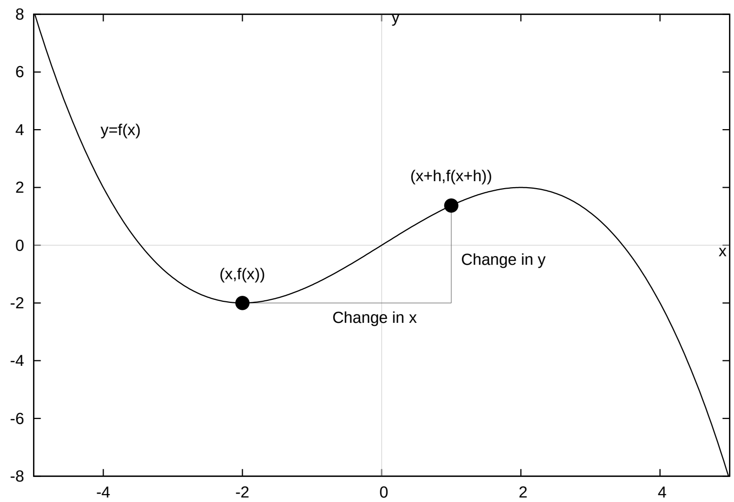
$$\Delta x = x_{new} - x_{old} \qquad \text{and} \qquad x_{new} = x_{old} + \Delta x.$$

Instead of using the *old* and *new* subscripts, we will often simply think about a change from x to $x + \Delta x$ (or from x to $x + h$).

If f is a function and $y = f(x)$, then

$$\Delta y = y_{new} - y_{old} = f(x_{new}) - f(x_{old}) = f(x + \Delta x) - f(x).$$

Throughout the course, we will be interested in the relationship between Δx and Δy.



Example 3 Let $y = g(x) = x^3 - 2x$. Simplify the expression for Δy.

$$\Delta y = g(x + \Delta x) - g(x) = \big[(x + \Delta x)^3 - 2(x + \Delta x)\big] - \big[x^3 - 2x\big]$$

$$= x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 2x - 2\Delta x - x^3 + 2x = 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 2\Delta x$$

Example 4 Let $y = f(x) = \dfrac{1}{x}$. Simplify the expression for Δy. Then simplify the expression for $\Delta y/\Delta x$.

$$\Delta y = f(x + \Delta x) - f(x) = \frac{1}{x + \Delta x} - \frac{1}{x} = \frac{x}{x(x + \Delta x)} - \frac{x + \Delta x}{x(x + \Delta x)} = \frac{-\Delta x}{x(x + \Delta x)}$$

$$\frac{\Delta y}{\Delta x} = \frac{1}{\Delta x}\left(\frac{-\Delta x}{x(x + \Delta x)}\right) = \frac{-1}{x(x + \Delta x)}$$

Graphing without the calculator

Even though we will often make use of the graphing calculator, it is important to have basic graphing skills. When we need to have a "rough" graph of a basic function, we can normally get it very quickly without the calculator.

Here is a short list of the graphing skills that we are all expected to have:

- Know the graphs of basic functions such as $y = mx + b$, $y = x^n$, $y = \sqrt{x}$, $y = |x|$, $y = \sin x$, $y = \cos x$, and $y = \tan x$.
- Know the general shape of the graph of a polynomial of degree n.
- Use x- and y-intercepts when graphing.

- Use vertical & horizontal translations and vertical & horizontal flips.
- Use symmetry.
  - A function is **even** if $f(-x) = f(x)$. The graph of an even function is symmetric about the y-axis.
  - A function is **odd** if $f(-x) = -f(x)$. The graph of an odd function is symmetric about the origin.
- Know a little bit about horizontal and vertical asymptotes.

Example 4 Explain how the graph of $y = (x+1)^2 - 3$ can be obtained from the graph of $y = x^2$.

Start with the graph of $y = x^2$ and shift it left 1 unit and down 3 units.

Example 5 Sketch the graph of $f(x) = (x-4)(x+2)$.

Solution omitted.

Example 6 Sketch the graph of $g(x) = |\sin 2\pi x|$.

Solution omitted.