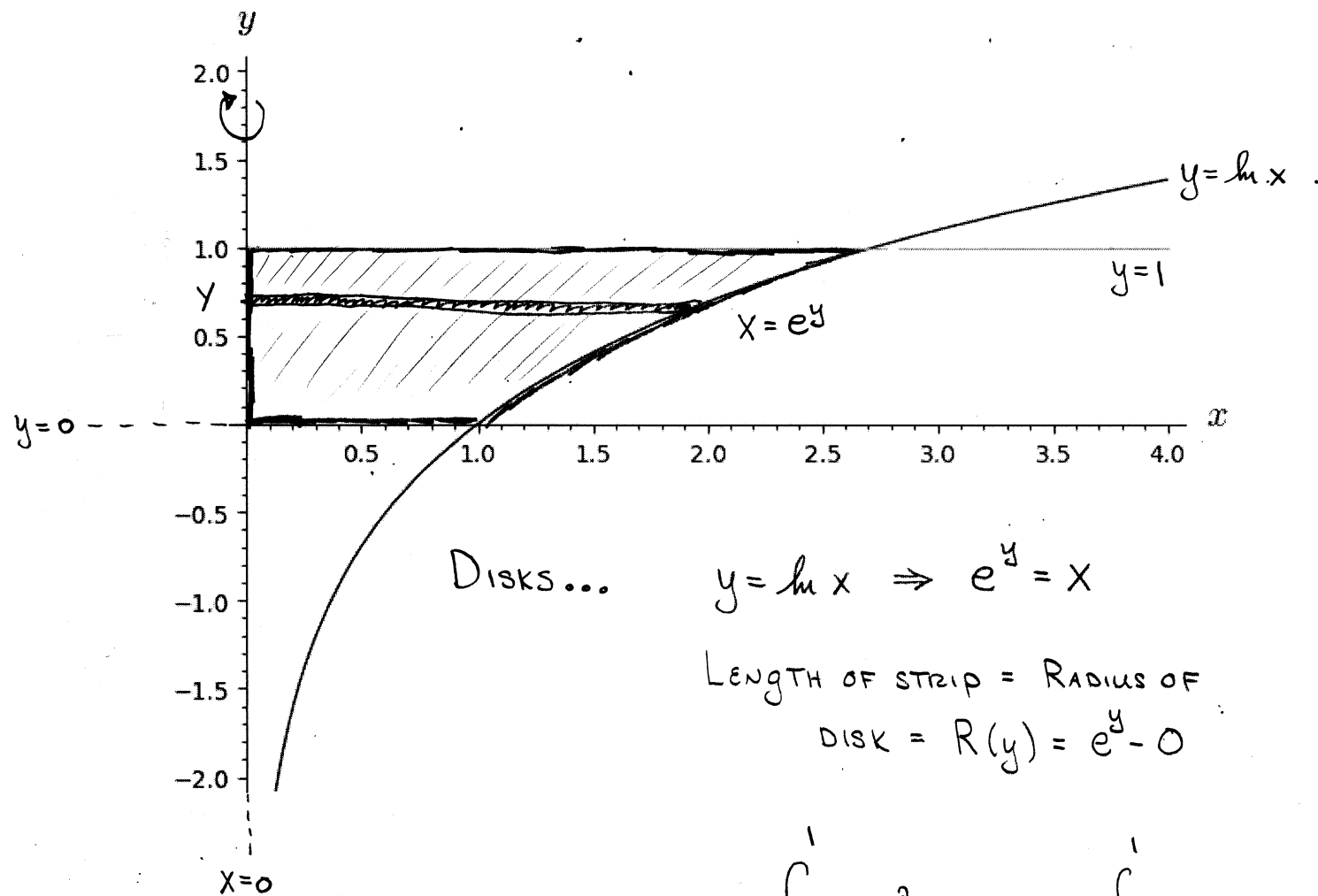


Example

The region bounded by the graphs of $y = \ln x$, $y = 1$, $y = 0$, and $x = 0$ is rotated about the y -axis. Find the volume of the solid that is generated.

Solution



$$\text{Volume} = \pi \int_0^1 (e^y)^2 dy = \pi \int_0^1 e^{2y} dy$$

$$\begin{aligned} \text{Volume} &= \frac{\pi}{2} \int_0^2 e^u du \\ &= \frac{\pi}{2} (e^u) \Big|_0^2 = \frac{\pi}{2} (e^2 - 1) \end{aligned}$$

Let $u = 2y$

$$du = 2 dy$$

$$\frac{1}{2} du = dy$$

$$y = 0 \Rightarrow u = 1$$

$$y = 1 \Rightarrow u = 2$$