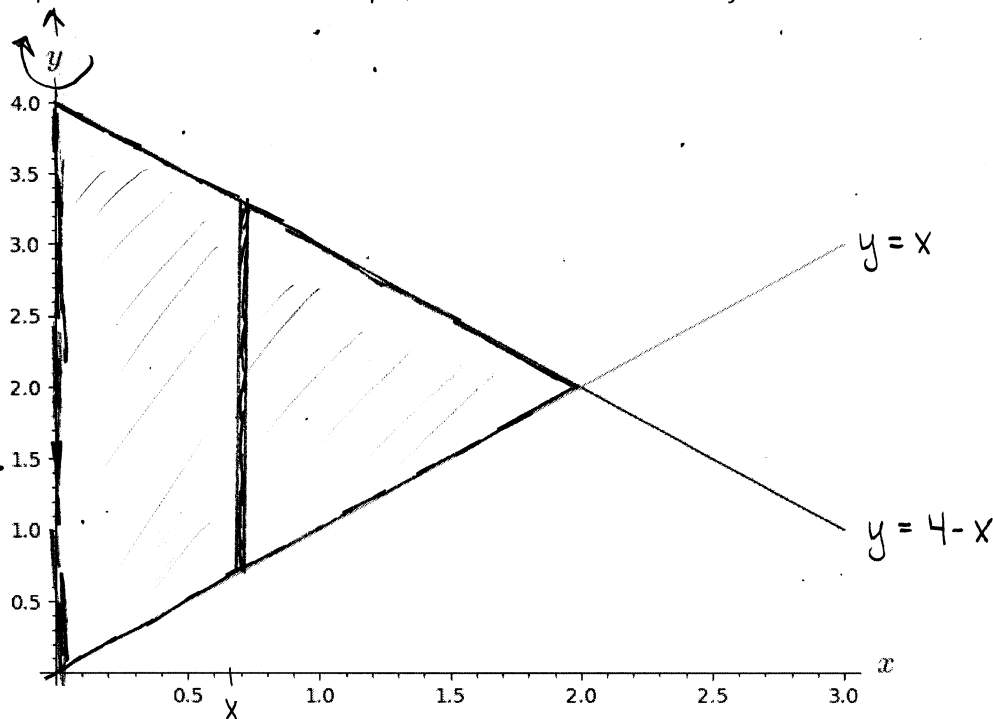


Example

The region bounded by the graphs of $y = 4 - x$, $y = x$, and $x = 0$ is rotated about the y -axis. Find the volume of the solid that is generated.

Solution

This is the same problem as an earlier example, but this time we will use cylindrical shells.



SHELLS...

$$4 - x = x \Rightarrow 4 = 2x \Rightarrow x = 2$$

STRIPS PILE UP FROM $x=0$ TO $x=2$

DISTANCE FROM ROTATION AXIS TO STRIP = x

$$\text{HEIGHT OF STRIP} = (4 - x) - x = 4 - 2x$$

$$\text{VOLUME} = 2\pi \int_0^2 x(4 - 2x) dx = 2\pi \int_0^2 (4x - 2x^2) dx$$

$$= 2\pi \left(2x^2 - \frac{2}{3}x^3 \right) \Big|_0^2 = 2\pi \left(8 - \frac{16}{3} \right)$$

$$= 2\pi \left(\frac{8}{3} \right) = \boxed{\frac{16\pi}{3}}$$