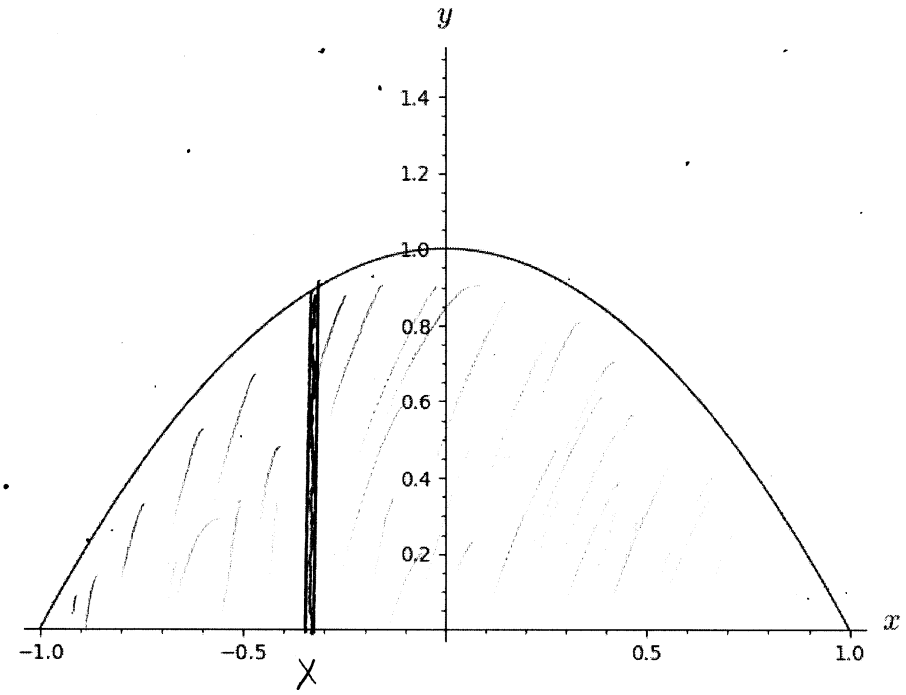
Example

The base of a solid is the region in the xy-plane that is bounded by the graphs of $y = 1 - x^2$ and $y = 0$. Cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

Solution



Strips pile up from $x=-1$ to $x=1$.

Area of cross section at x = (Height of strip)^2

$= (1-x^2)^2$

$$\text{Volume} = \int_{-1}^{1} (1-x^2)^2\,dx = \int_{-1}^{1} (1-2x^2+x^4)\,dx$$

$$= 2\int_{0}^{1} (1-2x^2+x^4)\,dx$$

$$= 2\left(x-\frac{2}{3}x^3+\frac{1}{5}x^5\right)\Big|_0^1 = 2\left(1-\frac{2}{3}+\frac{1}{5}\right)$$

$$= \frac{16}{15}$$