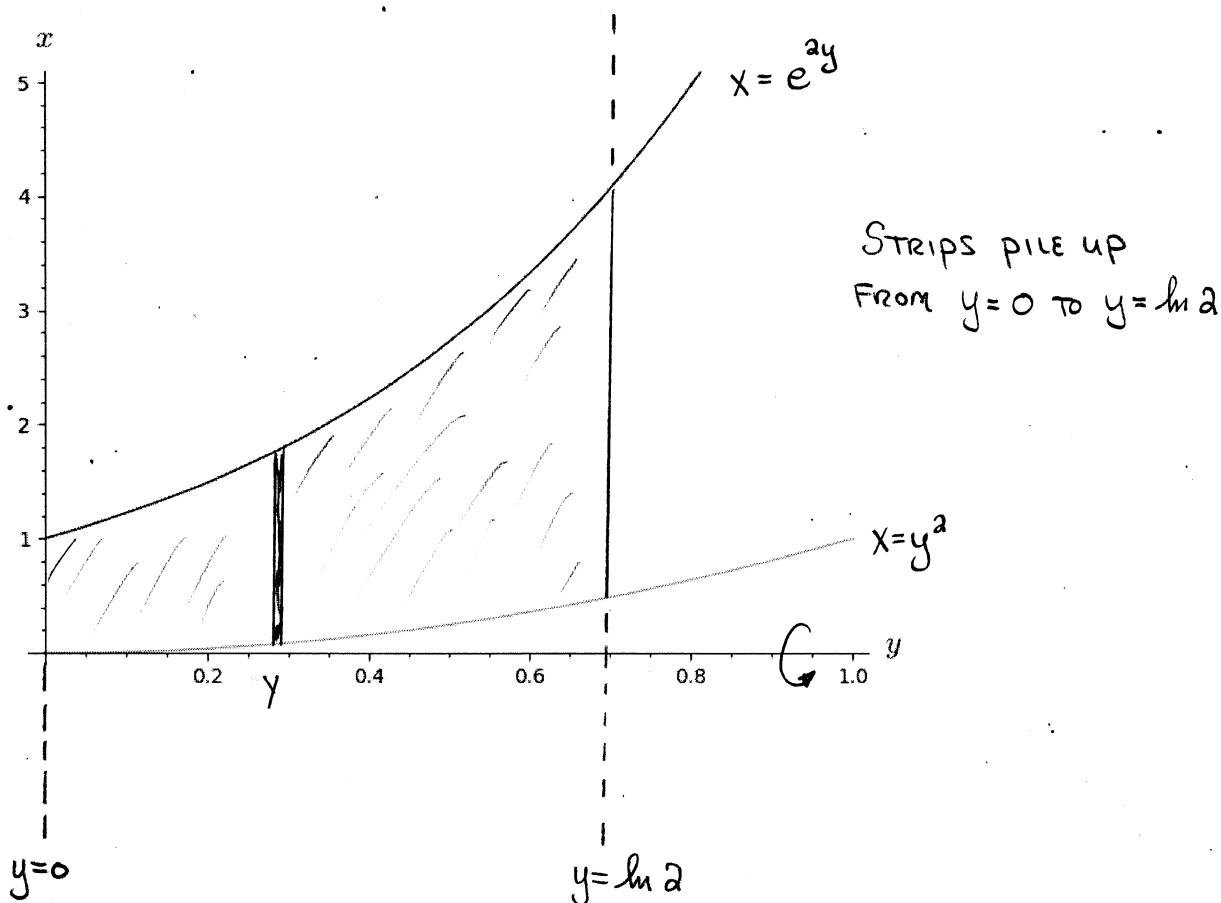# Example

The region bounded by the graphs of $x = e^{2y}$, $x = y^2$, $y = 0$, and $y = \ln 2$ is rotated about the y-axis. Find the volume of the solid that is generated.

Solution

In this problem, the independent variable is y. Therefore, I will place the y-axis horizontally and the x-axis vertically. An alternative approach is to rename the variables $x \leftrightarrow y$.

$x = e^{2y}$

STRIPS PILE UP
FROM $y = 0$ TO $y = \ln 2$

$X = y^2$

$y = 0$

$y = \ln 2$

WASHERS...

$$\text{VOLUME} = \pi \int_0^{\ln 2} \left[\left(e^{2y} \right)^2 - \left(y^2 \right)^2 \right] dy$$

$$= \pi \int_0^{\ln 2} \left(e^{4y} - y^4 \right) dy = \pi \left[\frac{1}{4} e^{4y} - \frac{1}{5} y^5 \right]_0^{\ln 2}$$

$$= \pi \left[\left(4 - \frac{(\ln 2)^5}{5} \right) - \left(\frac{1}{4} - 0 \right) \right] = \pi \left(\frac{15}{4} - \frac{(\ln 2)^5}{5} \right)$$

$$\approx 11.68044$$