# Math 109 - Review 2

October 14, 2019

Name key

These problems may help you review for Test 2. They are coded to match the course objectives from your syllabus. Your actual test will not be as long as this review packet. Unless otherwise indicated, you should simplify all answers by reducing fractions, simplifying radicals, and/or rationalizing denominators (as you've done on your ALEKS homework).

**Objective**: Find and apply the slope-intercept form of the equation of a line. [2]

1. Find the slope and $y$-intercept of the line described by $2x - 5y = 8$. Write your $y$-intercept as an ordered pair.

   $2x - 8 = 5y$

   $y = \frac{2}{5}x - \frac{8}{5}$

   Slope $= \frac{2}{5}$

   Y-INT $= \left(0, -\frac{8}{5}\right)$

2. A line with slope $-3/7$ has $y$-intercept $(0, -4)$. Find an equation of the line. Write your final answer in standard form.

   $y = -\frac{3}{7}x - 4 \Rightarrow 7y = -3x - 28$

   $\boxed{3x + 7y = -28}$

3. A line is described by the equation $y = 3x - 2$. Find the slope of the line, and determine two points on the line.

   Slope $= 3$

   Two points... $(0,-2)$, $(1,1)$

**Objective**: Apply the point-slope form of the equation of a line. [2]

4. A line with slope $3/5$ passes through the point $(10, 8)$. Find an equation for the line.

   $y - 8 = \frac{3}{5}(x - 10)$ OR $y = \frac{3}{5}x + 2$

5. A line is described by the equation $y + 2 = -4(x - 7)$. Find the slope of the line and a point on the line.

   Slope $= -4$

   Point $= (7, -2)$

   (CAN READ IT OFF THE EQUATION.)

6. A line passes through the two points $(-4, 3)$ and $(6, -2)$. Find an equation for the line.

$m = \frac{\Delta y}{\Delta x} = \frac{-5}{10} = -\frac{1}{2}$

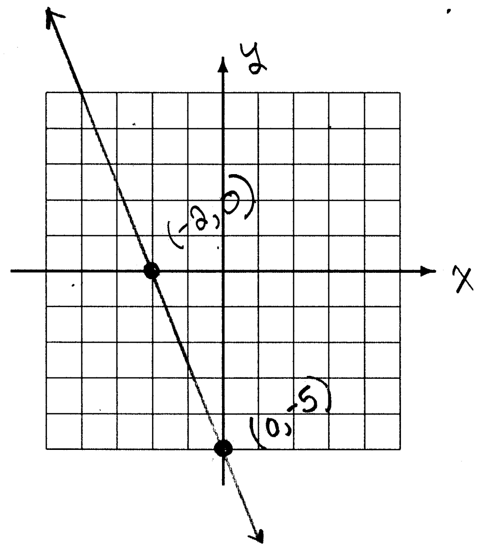
$y - 3 = -\frac{1}{2}(x+4)$ or $y = -\frac{1}{2}x + 1$

**Objective**: Graph a line using its slope and a point. [2]

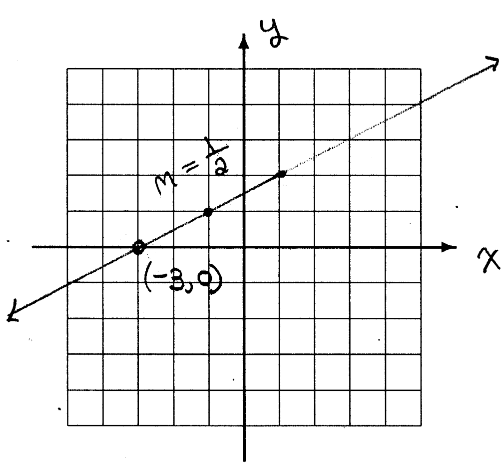
7. A line is described by the equation $5x + 2y = -10$. Rewrite the equation is slope-intercept form. Then use the intercept and the slope to sketch the graph. Be sure to label your axes.

$2y = -5x - 10$

$y = -\frac{5}{2}x - 5$




8. A line with slope 1/2 passes through the point $(-3, 0)$. Sketch the graph of the line. Be sure to label your axes.



**Objective**: Find lines parallel or perpendicular to given lines. [2]

9. A line passes through the points (1, 1) and (6, 4). Find an equation of the perpendicular line through (5, −6).

$m = \frac{\Delta y}{\Delta x} = \frac{3}{5}$

$m_{\perp} = -\frac{5}{3}$

$\boxed{y+6 = -\frac{5}{3}(x-5) \quad \text{OR} \quad y = -\frac{5}{3}x + \frac{7}{3}}$

10. A line passes through the point (−3, −2) and is parallel to the line described by $y = 8x - 7$. Find an equation of the line. Write your final answer in standard form.

$m = 8$

$m_{\parallel} = 8$

$y+2 = 8(x+3)$

$y+2 = 8x+24$

$\boxed{8x - y = -22}$

11. A line passes through the points (3, 1) and (3, 0). Find equations of the lines parallel and perpendicular to the original line. Label which is which.

VERTICAL

$x = 3$

$\boxed{x = 10 \ \text{--- PARALLEL} \quad y = 5 \ \text{--- PERP}}$ (VERT) (HORIZONTAL)

**Objective**: Apply lines and linear equations in real-world applications. [2]

12. The length of the humerus (the bone from the elbow to the shoulder) is a good indicator of height. A female with a humerus of length 26.1 cm is approximately 143.5 cm tall, while a female with a 20.4 cm humerus is about 127.6 cm tall. Assume that humerus length and height satisfy a linear equation. Determine that equation. Round all numbers to the nearest tenth.

(26.1, 143.5)
(20.4, 127.6)

$m = \frac{\Delta y}{\Delta x} = \frac{15.9}{5.7} \approx 2.8$

Using (26.1, 143.5) ...

$y - 143.5 = 2.8(x - 26.1)$

OR

$\boxed{y = 2.8x + 70.4}$ WHERE $x$ = HUMERUS LENGTH, $y$ = HEIGHT

13. A car currently worth \$24,575 depreciates at a constant rate of \$1752. Let $v$ represent the value of the car in dollars, and let $t$ represent time in years. Using the variables $v$ and $t$, write an equation for the value of the car.

$\boxed{V = 24575 - 1752t}$

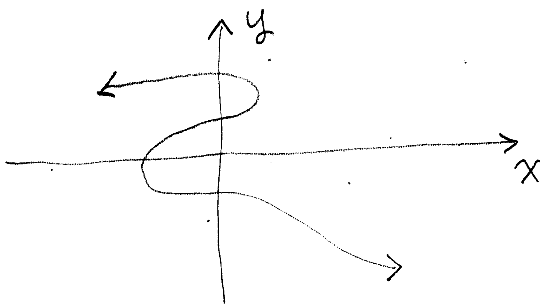
**Objective**: Determine whether a relation is a function. [1]

14. Carefully explain why this relation is not a function.

$$\{(1,2),(2,5),(3,8),(4,10),(-1,8),(3,9)\}$$

INPUT $X = 3$ HAS TWO DIFFERENT OUTPUTS.

15. Sketch the graph of a relation that is not a function.




FAILS VERTICAL LINE TEST

16. For any real number, $x$, let $f(x) = x^3 - x^2 + 1$. Does this define a function? How do you know?

YES, ONE X-INPUT GIVES EXACTLY ONE $f(x)$ - OUTPUT

17. Does this table describe a function? How do you know?

| $x$ | $-2$ | $2$ | $-5$ | $8$ | $7$ | $13$ |
|---|---|---|---|---|---|---|
| $y$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |

YES, NO X - COORDINATE IS REUSED.

**Objective**: Determine the domain and range of a function. [1]

18. What is the domain of the function $F(x) = x^2 + |x|$?

ALL REAL #'S ... $(-\infty, \infty)$

19. What is the domain of the function $g(x) = \frac{x^2 + x - 6}{x^2 + 6x + 5}$?

$\underbrace{x^2 + 6x + 5}$

$(x+5)(x+1) = 0$

$x = -5,\ x = -1$

$x \neq -5,\ x \neq -1 \ldots$

All Real #'s except $x = -5,\ x = -1$

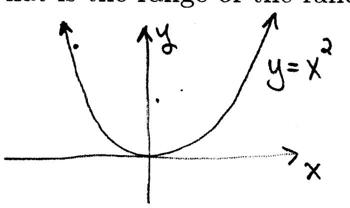
20. What is the domain and range of the relation defined by the following table of values?

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | $-2$ | 2 | $-5$ | 8 | 7 | 13 |
| $y$ | 1 | 1 | 1 | 1 | 1 | 1 |

Domain = $\{-2, 2, -5, 8, 7, 13\}$

Range = $\{1\}$

21. What is the range of the function $f(x) = x^2$?




$y \geq 0 \ldots [0, \infty)$

**Objective**: Use function notation and evaluate functions. [1,5]

22. Let $f(x) = \sqrt[4]{x + 7}$. Evaluate $f(9)$. What about $f(-8)$?

$f(9) = \sqrt[4]{16}$

$= 2$

$f(-8) = \sqrt[4]{-1}$ Not defined.

23. Let $g(y) = 2y^2 - 3y + 7$. Evaluate $g(-5)$.

$g(-5) = 2(-5)^2 - 3(-5) + 7$

$= 2(25) + 15 + 7 = 72$

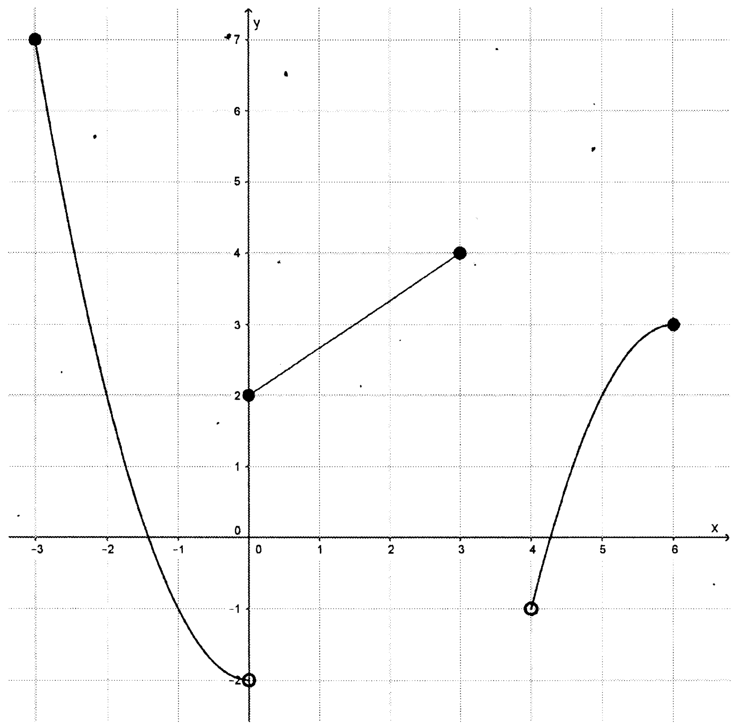
24. Let $G(x) = \frac{2x}{2x - 8}$. Evaluate $G(4)$.

$G(4)$ is not defined.

It would cause division by zero

**Objectives**: Interpret graphs of functions. Given the graph of a function, determine where the function is positive, negative, or zero; determine intervals on which the function is increasing, decreasing, or constant; and determine the local maxima and minima. [5]

25. The graph of $y = h(x)$ is shown below. Use the graph to solve each part of this problem.




(a) Is this the graph of a function? How do you know?

YES. THE GRAPH PASSES THE VERTICAL LINE TEST.

(b) What is the domain of $h$?

$[-3, 3] \cup (4, 6]$

(c) What is the range of $h$?

$(-2, 7]$

(d) Determine $h(-2)$.

$h(-2) \approx 2$

(e) Determine $h(3.5)$.

NOT DEFINED

(f) Determine $h(0)$.

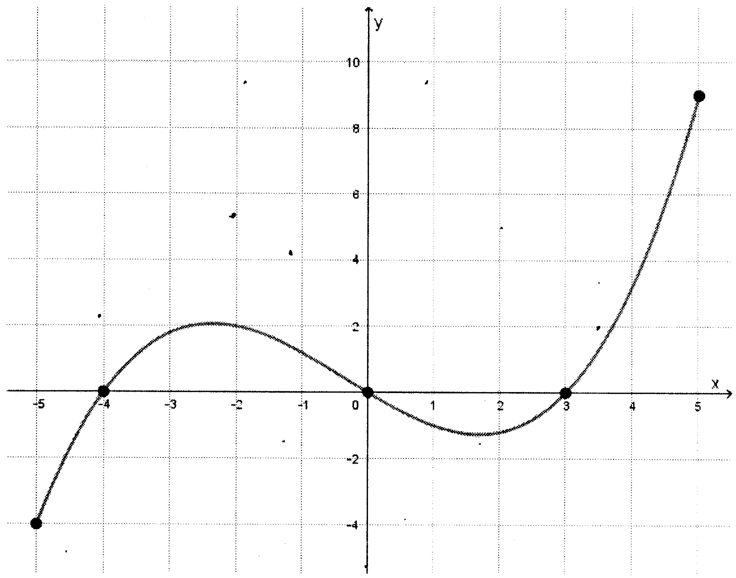
$h(0) = 2$

(g) Determine an $x$-value for which $h(x) = 3$. How many are there?

3 OF THEM...

$x \approx -2.25, \quad x \approx 1.5, \quad x = 6$

26. The graph of $y = f(x)$ is shown below.




(a) What is the domain of $f$?

$[-5, 5]$

(b) What is the range of $f$?

$[-4, 9]$

(c) Determine intervals on which $f(x) < 0$.

$[-5, -4) \cup (0, 3)$

(d) Determine intervals on which $f(x) > 0$.

$(-4, 0) \cup (3, 5]$

(e) Determine open intervals on which $f$ is increasing.

$(-5, -2.5) \cup (1.5, 5)$

(f) Determine open intervals on which $f$ is decreasing.

$(-2.5, 1.5)$

(g) Determine any relative (local) minimum values and maximum values.

REL MAX : $y = 2$ WHERE $x \approx -2.5$

REL MIN : $y \approx -1$ WHERE $x \approx 1.5$

**Objective**: Simplify difference quotients.

27. Let $f(x) = 5 - 2x$. Expand and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{[5-2(x+h)]-[5-2x]}{h} = \frac{5-2x-2h-5+2x}{h} = \frac{-2h}{h} = \boxed{-2}$$

28. Let $g(x) = x^2 - 3x + 7$. Expand and simplify the difference quotient $\dfrac{g(x+h)-g(x)}{h}$.

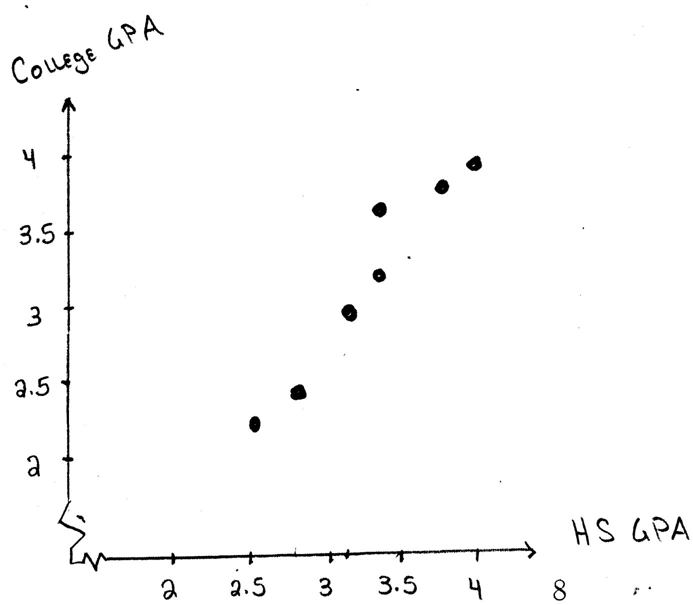
$$\frac{[(x+h)^2-3(x+h)+7]-[x^2-3x+7]}{h} = \frac{x^2+2xh+h^2-3x-3h+7-x^2+3x-7}{h}$$

$$= \frac{2xh+h^2-3h}{h} = \frac{h(2x+h-3)}{h} = \boxed{2x+h-3}$$

**Objective**: Construct a scatterplot. [4]

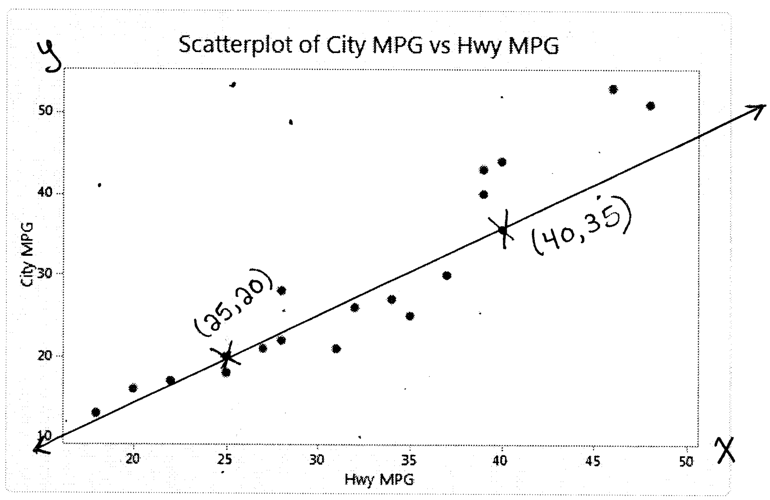
29. In the following $(x, y)$ ordered pairs, $x$ represents a student's high school GPA, and $y$ represents the same student's college GPA. Sketch the scatterplot.

$(3.2, 3.1)$, $(2.5, 2.2)$, $(3.6, 3.8)$, $(3.2, 3.6)$, $(3.1, 3.0)$, $(2.8, 2.4)$, $(4.0, 3.9)$



**Objective**: Recognize a linear relationship, and determine an equation that describes the relationship. [2,4]

30. A scatterplot is shown below. Sketch the best fit line. Then use two points on your line to determine an equation of the line. Write your answer in slope-intercept form.




Using (25,20) AND (40,35) ...

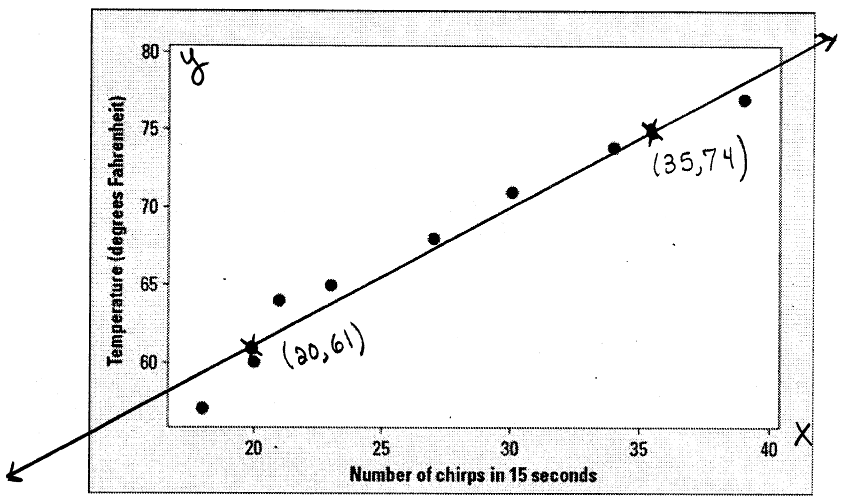
$m = \frac{15}{15} = 1$

$y - 20 = 1(x - 25)$

OR

$\boxed{y = x - 5}$

31. A scatterplot is shown below. Sketch the best fit line. Then use two points on your line to determine an equation of the line. Write your answer in slope-intercept form.




Using (20,61) AND (35,74) ...

$m = \frac{13}{15}$

$y - 61 = \frac{13}{15}(x - 20)$

$y = \frac{13}{15}x - \frac{260}{15} + \frac{915}{15}$

$\boxed{y = \frac{13}{15}x + \frac{655}{15}}$

OR $y \approx 0.87x + 43.67$

**Objective**: Use an equation to make predictions in a linear relationship. [2,4]

32. Look at the equation you determined in problem #30 above. Use your equation to predict the City MPG of a car whose Hwy MPG is 44.

$y = x - 5$

$y = 44 - 5 = \boxed{39}$

33. Look at the equation you determined in problem #30 above. Use your equation to predict the Hwy MPG of a car whose City MPG is 35.

$35 = x - 5$

$\Rightarrow \boxed{x = 40}$

34. Look at the equation you determined in problem #31 above. Use your equation to predict the temperature if there are 25 chirps in 15 minutes.

$y \approx 0.87x + 43.67$

$y = 0.87(25) + 43.67$

$\approx \boxed{65.4\,^\circ F}$

35. Look at the equation you determined in problem #31 above. Use your equation to predict the number of chirps in 15 minutes if the temperature is 70°F.

$70 = 0.87x + 43.67$

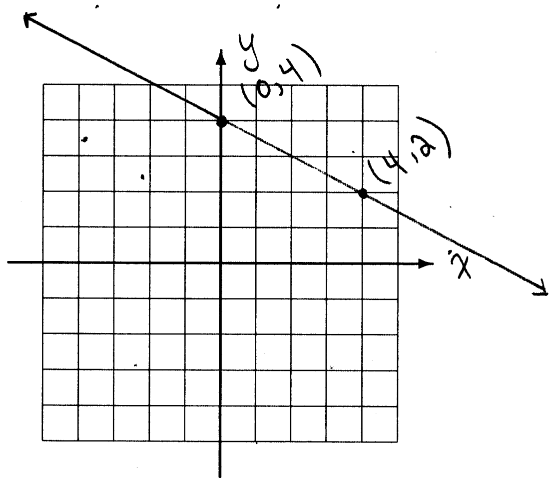
$x \approx 30.3$

About 30 chirps

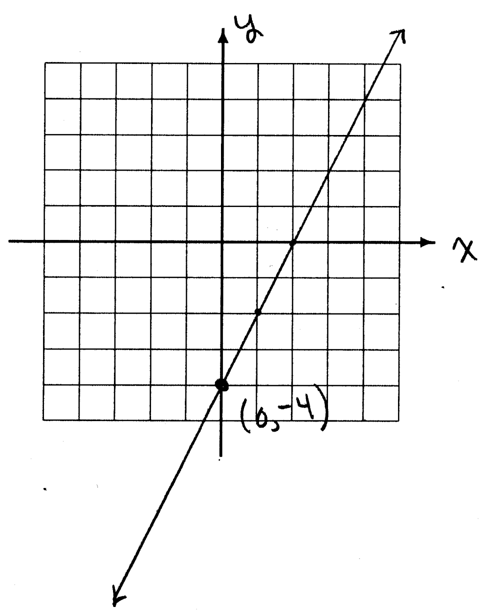
**Objective**: Sketch the graph of $f(x) = ax + b$. [2,3]

36. Determine two points on the graph of $f(x) = -\frac{1}{2}x + 4$. Then sketch the graph. Be sure to label your axes.

| x | y = f(x) |
|---|---|
| 0 | 4 |
| 4 | 2 |




37. Sketch the graph of $g(x) = 2x - 4$. Be sure to label your axes.



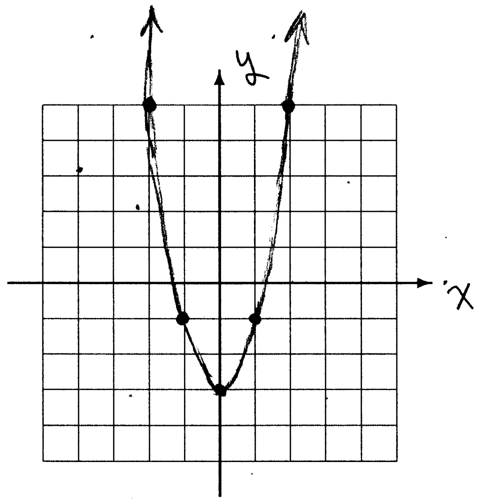

Slope 2,

Y-INT (0,-4

**Objective**: Sketch the graph of $f(x) = ax^2 + c$. [8]

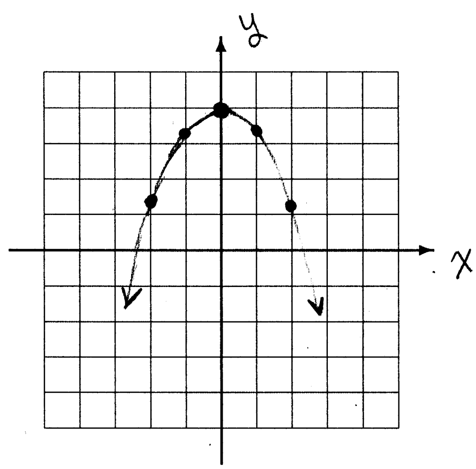
38. Determine five points on the graph of $f(x) = 2x^2 - 3$. Then plot your points and sketch the graph.

| $x$ | $y = f(x)$ |
|---|---|
| $0$ | $-3$ |
| $\pm 1$ | $-1$ |
| $\pm 2$ | $5$ |




39. Determine five points on the graph of $g(x) = 4 - \frac{2}{3}x^2$. Then plot your points and sketch the graph.

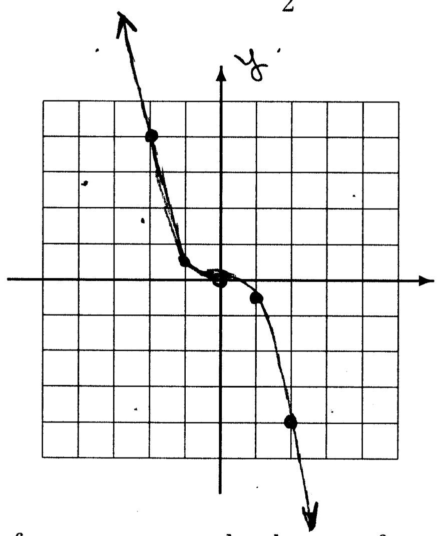
| $x$ | $y = g(x)$ |
|---|---|
| $0$ | $4$ |
| $\pm 1$ | $\frac{10}{3}$ |
| $\pm 2$ | $\frac{4}{3}$ |



**Objective**: Sketch the graph of $f(x) = ax^3$. [6]

40. Determine five points on the graph of $f(x) = -\frac{1}{2}x^3$. Then plot your points and sketch the graph.

| X | y = f(x) |
|---|---|
| 0 | 0 |
| 1 | $-\frac{1}{2}$ |
| -1 | $\frac{1}{2}$ |
| 2 | -4 |
| -2 | 4 |

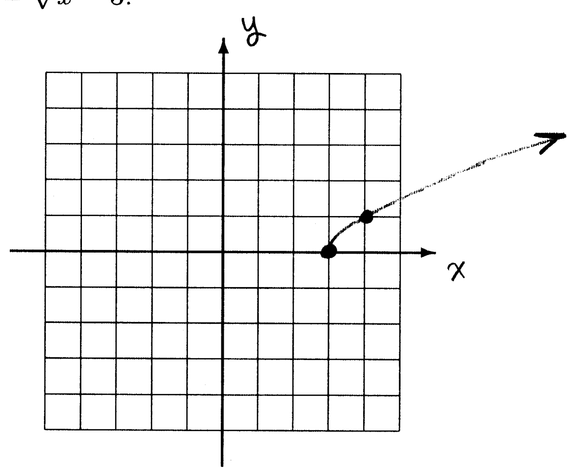



**Objective**: Sketch the graphs of square root and cube root functions. [6]

41. Sketch the graph of $f(x) = \sqrt{x-3}$.

$y = \sqrt{x}$

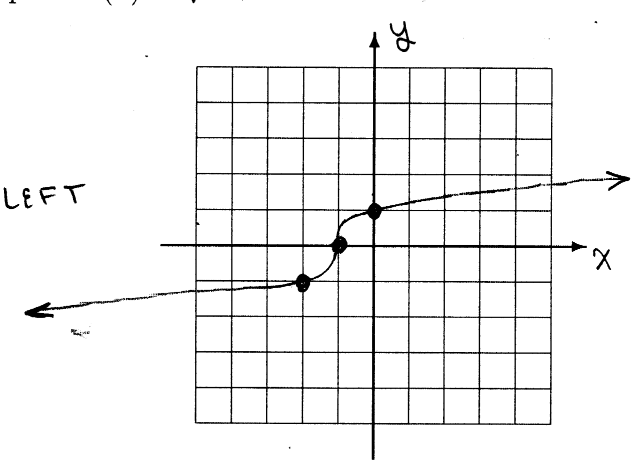
3 UNITS RIGHT




42. Sketch the graph of $h(x) = \sqrt[3]{x+1}$.

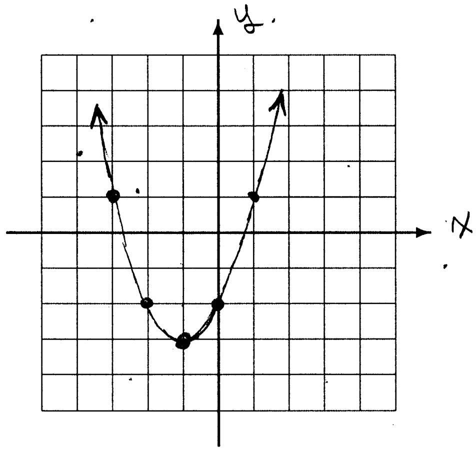
$y = \sqrt[3]{x}$

1 UNIT LEFT



**Objective**: Sketch the graphs of shifted parabolas. [6]

43. Sketch the graph of $g(x) = (x+1)^2 - 3$.




$y = x^2$ SHIFTED

1 LEFT

AND

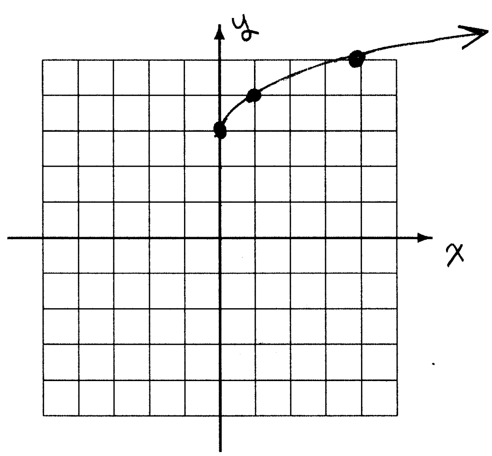
3 DOWN.

**Objective**: Use basic translations to sketch graphs. [6]

44. First describe the graph of $h(x) = \sqrt{x} + 3$. Then sketch it.

$y = \sqrt{x}$

up 3




45. What equation has the graph of $y = x^2$ shifted 8 units left and 2 units up?

$\boxed{y = (x+8)^2 + 2}$

46. Start with the graph of $y = \sqrt[3]{x}$. Shift it 4 units down, then 7 units right. What is an equation for the new graph?

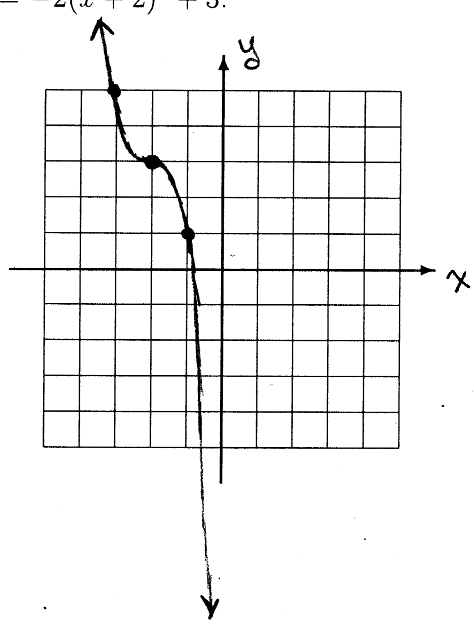
$$\boxed{y = \sqrt[3]{x-7} - 4}$$

47. Carefully describe the graph of $f(x) = (x-1)^2 + 2$.

THE GRAPH IS THE GRAPH OF $y = x^2$ SHIFTED RIGHT 1 UNIT AND UP 2 UNITS.

48. Sketch the graph of $g(x) = -2(x+2)^3 + 3$.

GRAPH OF $y = -2x^3$ SHIFTED LEFT 2 AND UP 3




| $x$ | $y = g(x)$ |
|---|---|
| -2 | 3 |
| -1 | 1 |
| -3 | 5 |
| 0 | -13 |