Math 109 - Review 3

November 11, 2019

Name key

These problems may help you review for Test 3. They are coded to match the course objectives from your syllabus. Your actual test will not be as long as this review packet. Unless otherwise indicated, you should simplify all answers by reducing fractions, simplifying radicals, and/or rationalizing denominators (as you've done on your ALEKS homework). Label your axes when graphing.

Objective: Transform a graph by reflecting about an axis. [6]

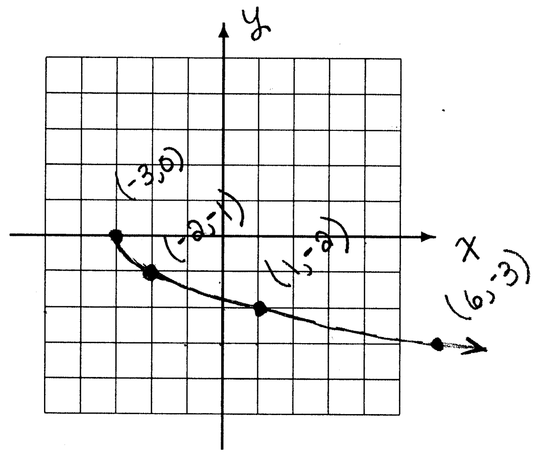
1. Explain how the graph of $y = -x^2$ can be obtained from the graph of $y = x^2$.

REFLECT THE GRAPH OF $y = x^2$ ABOUT THE X-AXIS TO OBTAIN THE GRAPH OF $y = -x^2$.

2. Explain how the graph of $f(x) = -\sqrt{x+3}$ is related to the graph of $g(x) = \sqrt{x+3}$. Then sketch the graph of f.

THE GRAPH OF f IS THE GRAPH OF g REFLECTED ABOUT THE X-AXIS.

$\uparrow$ $y = \sqrt{x}$ LEFT 3



Objective: Transform a graph by compressing or stretching. [6]

3. Explain how the graph of $y = 5x^2$ is related to the graph of $y = x^2$.

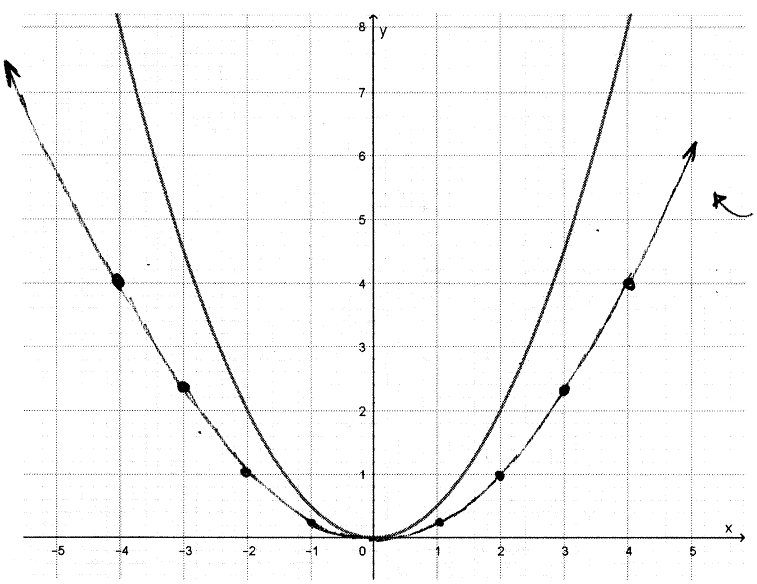
VERTICALLY STRETCH THE GRAPH OF $y = x^2$ BY A FACTOR OF 5 TO OBTAIN THE GRAPH OF $y = 5x^2$... EVERY NEW y-VALUE IS 5 TIMES THE OLD y-VALUE.

4. Explain how the graph of $y = \frac{1}{8}x^3$ is related to the graph of $y = x^3$.

VERTICALLY COMPRESS THE GRAPH OF $y = x^3$ TO OBTAIN THE GRAPH OF $y = \frac{1}{8}x^3$... EVERY NEW y-VALUE IS $\frac{1}{8}$ THE OLD y-VALUE.

5. The graph of $y = f(x)$ is shown below. In the same coordinate system, sketch the graph of $y = \frac{1}{2}f(x)$.

EACH NEW y IS HALF THE OLD y.



$y = \frac{1}{2}f(x)$

Objective: Carry out a sequence of transformations on a graph. [6]

6. Describe the sequence of transformations that transform the graph of $y = x$ to that of $y = (x + 4) - 5$.

① SHIFT LEFT 4 UNITS

② SHIFT DOWN 5 UNITS

7. Describe the sequence of transformations that transform the graph of $y = x^3$ to that of $y = -(x - 2)^3 + 8$.

① SHIFT RIGHT 2 UNITS

② REFLECT ABOUT X-AXIS

③ SHIFT UP 8 UNITS

8. How is the graph of $y = \sqrt{x}$ related to the graph of $y = 5\sqrt{x + 1}$?

THE GRAPH OF $y = 5\sqrt{x+1}$ IS THE GRAPH OF $y=\sqrt{x}$ SHIFTED 1 UNIT LEFT AND VERTICALLY STRETCHED BY A FACTOR OF 5.

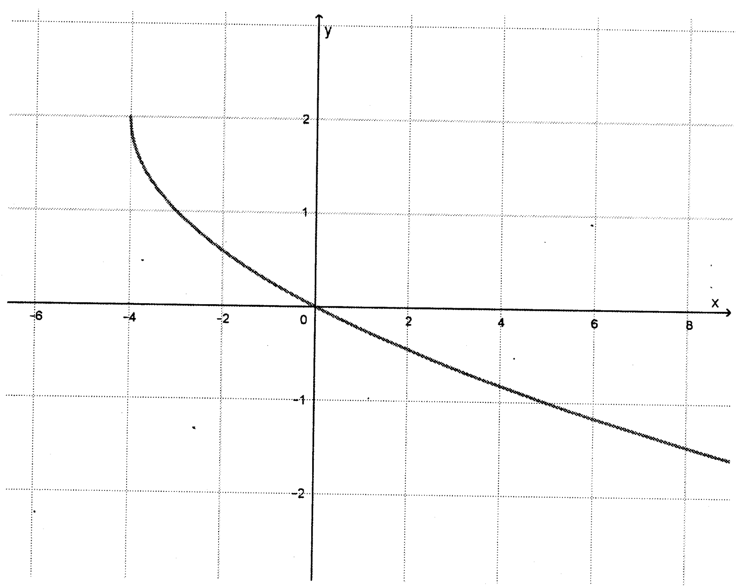
9. The graph of $y = \sqrt{x}$ is transformed by shifts and reflections to obtain the new graph shown below. What is an equation for the new graph?

START WITH $y=\sqrt{x}$

① SHIFT LEFT 4

② REFLECT ABOUT X-AXIS

③ SHIFT UP 2



$$\boxed{y = -\sqrt{x+4} + 2}$$

10. **Carefully sketch** the graph of each function. Your graph should show details such as correct scale and position. (Label your axes.)

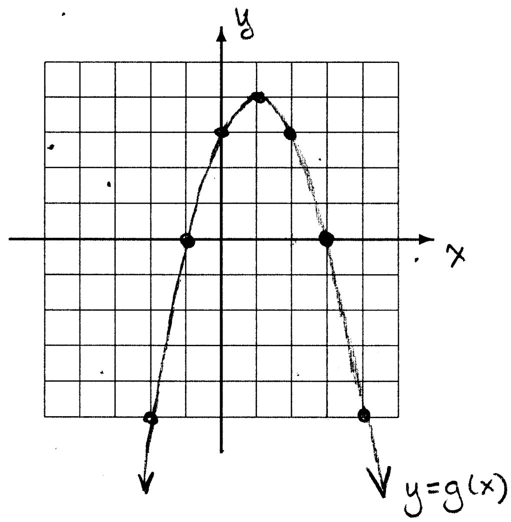
(a) $g(x) = 4 - (x-1)^2$

$y = x^2$

① 1 RIGHT

② REFLECT VERTICALLY

③ UP 4

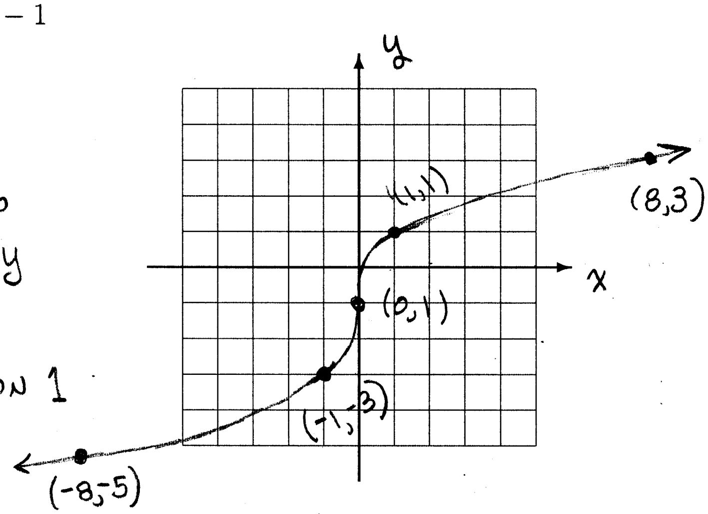


(b) $f(x) = 2\sqrt[3]{x} - 1$

$y = \sqrt[3]{x}$

① STRETCH TO DOUBLE y COORDS

② SHIFT DOWN 1



Objective: Compute sums, differences, products, and quotients of functions. [6]

11. Let $f(x) = 3x^2 - 8x + 1$ and $g(x) = x^2 + 8x - \sqrt{x}$.

(a) Find and simplify a formula for $(f+g)(x)$.

$$(f+g)(x) = (3x^2 - 8x + 1) + (x^2 + 8x - \sqrt{x})$$
$$= \boxed{4x^2 + 1 - \sqrt{x}}$$

(b) Evaluate $(f+g)(4)$.

$$(f+g)(4) = 4(4)^2 + 1 - \sqrt{4} = 64 + 1 - 2 = \boxed{63}$$

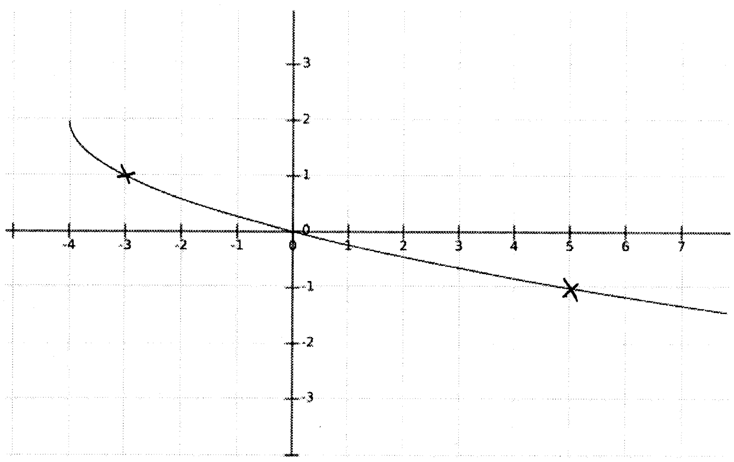
(c) Determine the domain of $f+g$.

$$(f+g)(x) = 4x^2 + 1 - \sqrt{x}$$

$x \geq 0$

DOMAIN = $[0, \infty)$

12. Let $g(x) = 3x - 5$. Using the function g and the graph of $y = f(x)$ shown below, compute each of the following.



(a) $(g-f)(5) = g(5) - f(5) = (3(5)-5) - (-1) = 10 + 1 = \boxed{11}$

(b) $(f+g)(-3) = f(-3) + g(-3) = (3(-3) - 5) + 1 = -14 + 1$
$= \boxed{-13}$

13. Some values of the functions f and g are given in the table below. Use the data from the table to evaluate each of the following.

x	0	1	2	3	4
$f(x)$	2	4	7	1	0
$g(x)$	5	0	4	8	3

(a) $(g-f)(4)$ $g(4) - f(4) = 3 - 0 = \boxed{3}$

(b) $(fg)(3)$ $f(3) \times g(3) = (1)(8) = \boxed{8}$

(c) $\left(\frac{g}{f}\right)(1)$ $\frac{g(1)}{f(1)} = \frac{0}{4} = \boxed{0}$

(d) $(f+f)(4)$ $f(4) + f(4) = 0 + 0 = \boxed{0}$

(e) $\left(\frac{f}{g}\right)(1)$ $\frac{f(1)}{g(1)} = \frac{4}{0}$ $\boxed{\text{NOT DEFINED}}$

Objective: Compute a composition of functions. [6]

14. Refer to the functions f and g defined in the problem above.

(a) Make a table of values for the function $(g \circ f)$.

X	0	1	2	3	4
$(g \circ f)(x)$ $g(f(x))$	4	3	DNE	0	5

(b) What is the domain of $(g \circ f)$?

$\{0, 1, 3, 4\}$

(c) What is the range of $(g \circ f)$?

$\{0, 3, 4, 5\}$

15. Let $f(x) = \sqrt{x}$ and $g(x) = 2x + 1$. Evaluate each of the following.

(a) $(f \circ g)(4)$ $= f(g(4)) = f(9) = \sqrt{9} = \boxed{3}$

(b) $(g \circ f)(9)$

$= g(f(9)) = g(3) = \boxed{7}$

16. Let $f(x) = \sqrt{2x-4}$ and $g(x) = 3x+5.$

(a) Find and simplify the formula for $(f \circ g)(x)$.

$$f(g(x)) = \sqrt{\underbrace{2(3x+5)-4}_{6x+10}} = \boxed{\sqrt{6x+6}}$$

(b) What is the domain of $(f \circ g)$?

$6x+6 \geq 0$

$6x \geq -6$

$x \geq -1$

$\boxed{\text{DOMAIN IS } [-1,\infty)}$

Objective: Write a function as a composition of functions. [6]

17. Find two functions f and g so that $(f \circ g)(x) = \sqrt{x^2+x+1}$.

$\boxed{f(x) = \sqrt{x},\ g(x) = x^2+x+1}$

18. Find two functions f and g so that $(f \circ g)(x) = (2x+1)^5 + 7(2x+1)^3$.

$f(x) = x^5 + 7x^3$

$g(x) = 2x+1$

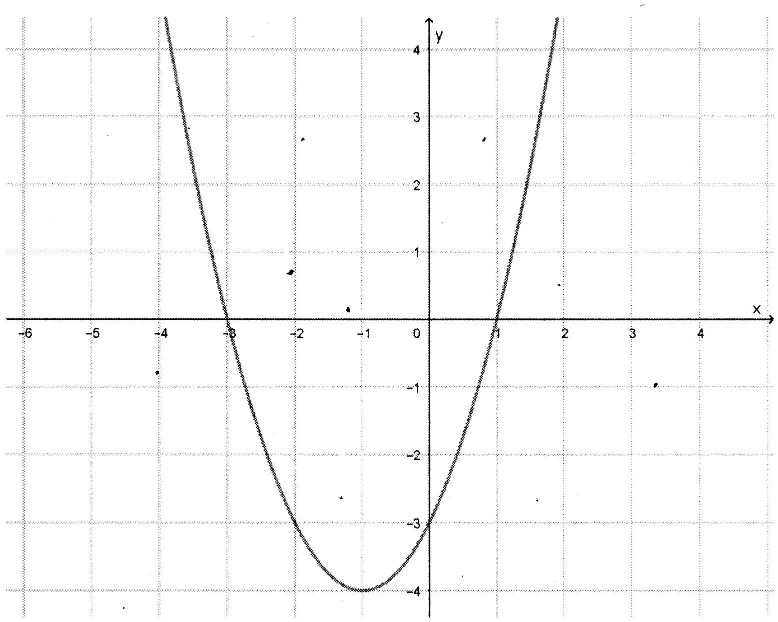
Objective: Find the vertex, intercepts, and symmetry axis of a parabola. [5,7,8]

19. The graph of $y = (x+5)^2 - 8$ is a parabola. Where is its vertex? What is an equation of the symmetry axis?

VERTEX AT $(-5,-8)$

SYMMETRY AXIS : $X=-5$

20. The graph of the function f is the parabola shown below.



(a) Is the leading coefficient of $f(x)$ positive or negative? How do you know?

Positive, graph opens up.

(b) Determine the vertex of the parabola.

$(-1,-4)$

(c) Determine the x-intercepts of the graph. Write them as ordered pairs.

$(-3,0)$ & $(1,0)$

(d) Determine the y-intercept of the graph. Write it as an ordered pair.

$(0,-3)$

(e) Write an equation for the axis of symmetry of the parabola.

$x=-1$

21. A quadratic function has a leading coefficient of 3 and zeros $x = 6$ and $x = -2$. Write its equation in standard form. What are the coordinates of vertex of its graph?

$f(x) = 3(x-6)(x+2) = 3(x^2-4x-12)$

$\boxed{f(x) = 3x^2 - 12x - 36}$

Vertex... $x = \frac{6+(-2)}{2} = 2$

$y = f(2) = -48$

$\boxed{(2,-48)}$

22. The graph of $f(x) = -2(x-3)^2 + 4$ is a parabola.

(a) Does the parabola open up or down? How do you know?

$\boxed{\text{Down, } a = -2 < 0}$

(b) Determine the vertex of the parabola.

Vertex at $\boxed{(3,4)}$

(c) Write an equation for the symmetry axis.

$\boxed{x = 3}$

Objective: Write a quadratic function in vertex form. [8]

23. Write the quadratic function $f(x) = x^2 + 6x + 11$ in vertex form.

Complete square

$f(x) = x^2 + 6x + 9 \quad + 2$

$\boxed{f(x) = (x+3)^2 + 2}$

24. Write the quadratic function $g(x) = x^2 - 4x - 2$ in vertex form.

$g(x) = x^2 - 4x + 4 \quad - 6$

$\boxed{g(x) = (x-2)^2 - 6}$

Objective: Find the zeros of a polynomial and determine their multiplicities. [9]

25. Find the zeros of $f(x) = x^2 + 3x + 2$. $(x+1)(x+2) = 0$

$x = -1$, mult 1
$x = -2$, mult 1

26. Find the zeros of $g(x) = 4(x-1)^3(x+5)(x+8)^2$ and state their multiplicities.

$x = 1$, mult 3
$x = -5$, mult 1
$x = -8$, mult 2

27. Let $f(x) = x^7(x-1)^5(x+2)$. Find the zeros of f and their multiplicities.

$x = 0$, mult 7
$x = 1$, mult 5
$x = -2$, mult 1

Objective: Carry out polynomial long division and synthetic division. [9]

28. Use long division to divide: $\dfrac{3x^2 + 3x - 14}{x - 2}$

$3x + 9$

$x - 2 \,\overline{)\,3x^2 + 3x - 14}$

$-(3x^2 - 6x)$

$9x - 14$

$-(9x - 18)$

4

$$\boxed{3x + 9 + \frac{4}{x-2}}$$

29. Use long division to divide: $(6x^3 \div 5x^2 - 3) \div (3x + 2)$

$2x^2 - 3x + 2$

$3x + 2 \,\overline{)\,6x^3 - 5x^2 + 0x - 3}$

$-(6x^3 + 4x^2)$

$-9x^2 + 0x - 3$

$-(-9x^2 - 6x)$

$6x - 3$

$-(6x + 4)$

-7

$$\boxed{2x^2 - 3x + 2 - \frac{7}{3x+2}}$$

30. Use long division to divide: $(8x^3 - 6x^2 - 11x + 13) \div (2x^2 - x)$

$4x - 1$

$2x^2 - x \,\overline{)\,8x^3 - 6x^2 - 11x + 13}$

$-(8x^3 - 4x^2)$

$-2x^2 - 11x + 13$

$-(-2x^2 + x)$

$-12x + 13$

$$\boxed{4x - 1 + \frac{-12x + 13}{2x^2 - x}}$$

31. Use long division to divide: $(-11 + 12x^2 + 5x + 9x^3) \div (-3x^2 - 2x + 2)$

$-3x - 2$

$-3x^2 - 2x + 2 \,\overline{)\,9x^3 + 12x^2 + 5x - 11}$

$-(9x^3 + 6x^2 - 6x)$

$6x^2 + 11x - 11$

$-(6x^2 + 4x - 4)$

$7x - 7$

$$\boxed{-3x - 2 + \frac{7x - 7}{-3x^2 - 2x + 2}}$$

32. Use synthetic division to divide: $\dfrac{3x^2 + 3x - 14}{x - 2}$

$$\begin{array}{r|rrr} 2 & 3 & 3 & -14 \\ & & 6 & 18 \\ \hline & 3 & 9 & 4 \end{array}$$

$$\boxed{3x + 9 + \frac{4}{x-2}}$$

33. Use synthetic division to divide: $(3x^4 + 7x^3 - 5x^2 + x - 6) \div (x + 3)$

$$\begin{array}{r|rrrrr} -3 & 3 & 7 & -5 & 1 & -6 \\ & & -9 & 6 & -3 & 6 \\ \hline & 3 & -2 & 1 & -2 & 0 \end{array}$$

$$\boxed{3x^3 - 2x^2 + x - 2}$$

34. Use synthetic division to divide: $\dfrac{x^3 - 21x + 20}{x + 5}$

$$\begin{array}{r|rrrr} -5 & 1 & 0 & -21 & 20 \\ & & -5 & 25 & -20 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

$$\boxed{x^2 - 5x + 4}$$

Objective: Apply the remainder and factor theorems. [9]

35. Use synthetic division and the remainder theorem to evaluate $P(-2)$ if $P(x) = -2x^4 - 2x^3 + x + 3$.

$$\begin{array}{r|rrrrr} -2 & -2 & -2 & 0 & 1 & 3 \\ & & 4 & -4 & 8 & -18 \\ \hline & -2 & 2 & -4 & 9 & -15 \end{array}$$

$\boxed{P(-2) = -15}$

36. Use synthetic division and the remainder theorem to evaluate $f(2)$ if $f(x) = 2x^2+3x+1$.

$$\begin{array}{r|rrr} 2 & 2 & 3 & 1 \\ & & 4 & 14 \\ \hline & 2 & 7 & 15 \end{array}$$

$\boxed{f(2) = 15}$

37. Let $f(x) = x^3 - 2x^2 - 4x + 8$. Evaluate $f(2)$. Based on the value of $f(2)$, determine whether $x - 2$ is a factor of f.

$f(2) = 8 - 8 - 8 + 8 = 0 \Rightarrow$ $\boxed{x - 2 \text{ IS A FACTOR}}$

38. Let $f(x) = 4x^3 - 3x^2 - 2x + 6$. Evaluate $f(-1)$. Based on the value of $f(-1)$, determine whether $x + 1$ is a factor of f.

$f(-1) = -4 - 3 + 2 + 6 = 1 \Rightarrow$ $\boxed{x + 1 \text{ IS NOT A FACTOR}}$

39. The only zeros of a polynomial are $x = 0$, $x = 5$ and $x = -8$. Determine the factors of the polynomial.

$x(x-5)(x+8)$

Objective: Determine the end behavior of a polynomial function. [10]

40. Describe the end behavior of the graph of $f(x) = -4x^8 - 19x^5 + 52x^2 - 17x + 100$.

$-4x^8 \rightarrow$ DOWN LEFT / DOWN RIGHT

41. Describe the end behavior of the graph of $f(x) = x^3 - 3x^2 - 9x - 17$.

$x^3 \rightarrow$ DOWN LEFT / UP RIGHT

42. Describe the end behavior of the graph of $f(x) = -3x^2(x+1)(x^2+1)$.

$-3x^5 \rightarrow$ UP LEFT / DOWN RIGHT

Objective: Use intercepts and end behavior to graph a polynomial function. [9,10]

43. Consider the polynomial $f(x) = -x(x-2)^3(2x+1)^2$.

(a) Determine the degree of f and the leading coefficient.

LEADING TERM $= (-x)(x)^3(2x)^2 = -4x^6 \rightarrow$ DEGREE IS 6. LEADING COEFF IS -4.

(b) State the zeros of f and their corresponding multiplicities.

$x = 0$, MULT 1

$x = 2$, MULT 3

$x = -\frac{1}{2}$, MULT 2

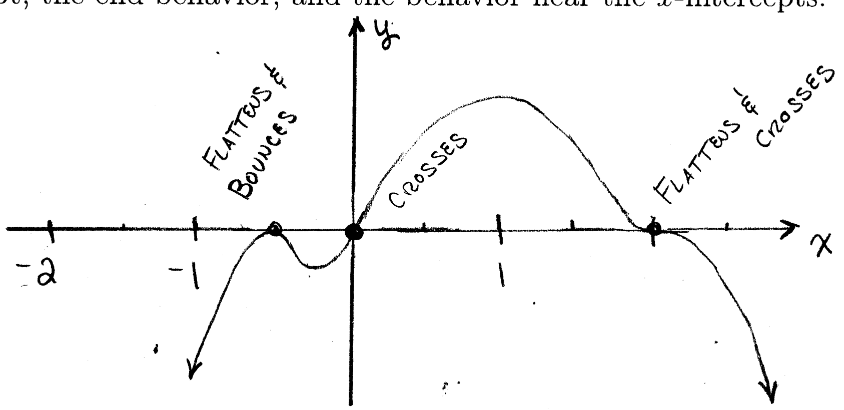
(c) Describe the end behavior of the graph of f.

$-4x^6 \rightarrow$ DOWN LEFT / DOWN RIGHT

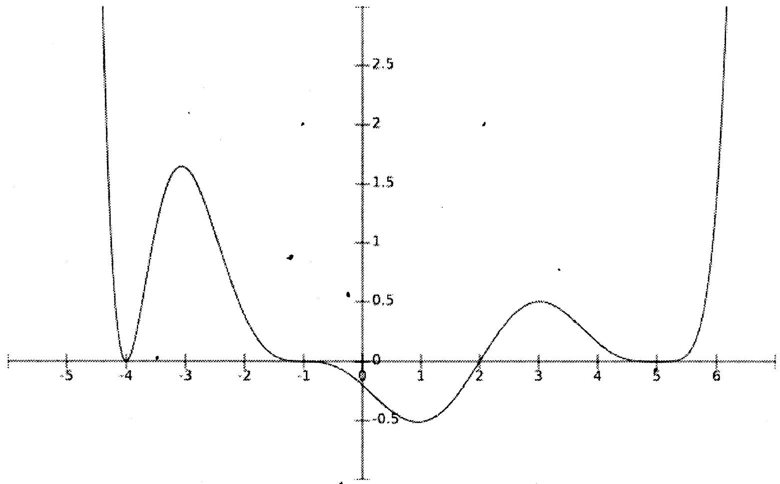
(d) Determine the y-intercept.

$x = 0 \Rightarrow f(0) = -(0)(-2)^3(1)^2 = 0$ $(0,0)$

(e) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the y-intercept, the end behavior, and the behavior near the x-intercepts.



44. The graph of a polynomial is shown below.



(a) Is the degree even or odd? Up LEFT / Up RIGHT $\Rightarrow$ EVEN DEGREE

(b) Is the leading coefficient positive or negative? POSITIVE

(c) Which zeros have multiplicity one?

$X = 2$

(d) Which zeros have even multiplicity?

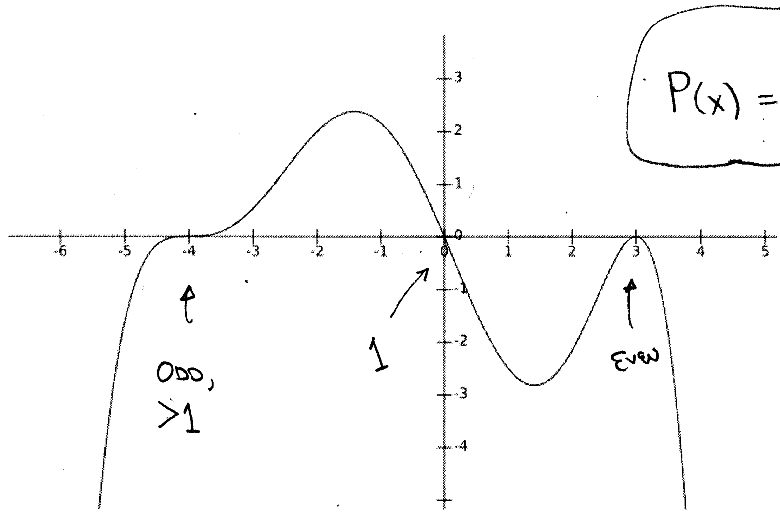
$X = -4, \ X = 5$

(e) Which zeros have odd multiplicity greater than 1?

$X = -1$

45. Give the factored form of a polynomial whose graph has the same general shape of the one given below.

$P(x) = (x+4)^3 (x)(x-3)^2$



Objective: Determine the vertical, horizontal, and/or slant asymptotes of the graph of a rational function. [10,11]

46. Let $f(x) = \dfrac{x^2+1}{(x-3)(x+2)}$. Determine the vertical asymptotes of the graph of f.

$\boxed{\begin{array}{c} X=3 \\ X=-2 \end{array}}$

THESE MAKE THE DENOM ZERO,

BUT NOT THE NUMERATOR.

47. Let $g(x) = \dfrac{2x-2}{(x-1)(x-5)}$. Determine the vertical asymptotes of the graph of g.

$$\frac{2(X-1)}{(X-1)(X-5)} = \frac{2}{X-5}$$

$\boxed{X=5}$

48. Let $f(x) = \dfrac{x^3+3x^2+1}{x^2-x}$. Determine the slant asymptote and the vertical asymptotes of the graph of f.

$$\begin{array}{r} X+4 \\ X^2-X \overline{\smash{)}\, X^3+3X^2+0X+1} \\ -(X^3-X^2) \\ \hline 4X^2+0X+1 \\ -(4X^2-4X) \\ \hline 4X+1 \end{array}$$

$$f(x) = X+4+\frac{4X+1}{X(X-1)}$$

SLANT ASYMP: $y = X+4$

V. A. $X=0$
$X=1$

$X^2-X = X(X-1)$

ZEROS OF DENOM ARE

$X=0,\ X=1$

49. Determine the horizontal asymptote of the graph of $R(x) = \dfrac{3x^2 + 2x - 9}{2x^2 - x - 1}$.

DEGREE SAME

$$\boxed{y = \frac{3}{2}}$$

50. Determine the horizontal asymptote of the graph of $H(x) = \dfrac{3x}{6x^2 + 3x + 112}$.

DEGREE TOP LESS THE DEGREE BOTTOM

$$\boxed{y = 0}$$

51. Explain how you know that the graph of $f(x) = \dfrac{x^3 - 8}{x + 7}$ has no horizontal asymptote.

THE DEGREE OF THE NUMERATOR (3) IS GREATER THAN THE DEGREE OF THE DENOMINATOR (1).

52. Give an example of a rational function whose graph has the horizontal asymptote $y = 3$ and vertical asymptotes $x = 5$ and $x = -9$.

$$R(x) = \frac{3x^2}{(x-5)(x+9)}$$

Objective: Sketch the graph of a rational function. [10,11]

53. Sketch the graph of $y = \dfrac{2x-4}{x-5}$. $= \dfrac{2(x-2)}{x-5}$

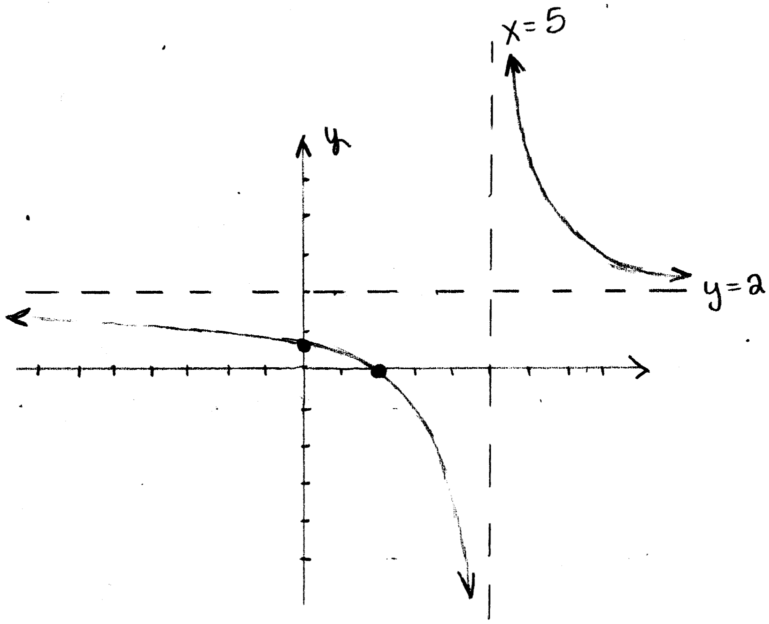
H.A. $y = 2$

V.A. $x = 5$

X-INTERCEPT: $(2,0)$

Y-INTERCEPT: $\left(0, \frac{4}{5}\right)$

PLOT SOME POINTS & USE GRAPHING CALC.



54. Sketch the graph of $y = \dfrac{x^2-2x-3}{2x+6}$.

$$\begin{array}{r} \tfrac{1}{2}x - \tfrac{5}{2} \\ 2x+6 \enclose{longdiv}{x^2 - 2x - 3} \\ -(x^2+3x) \\ \hline -5x-3 \\ -(-5x-15) \\ \hline 12 \end{array}$$

SLANT ASYMP: $y = \frac{1}{2}x - \frac{5}{2}$

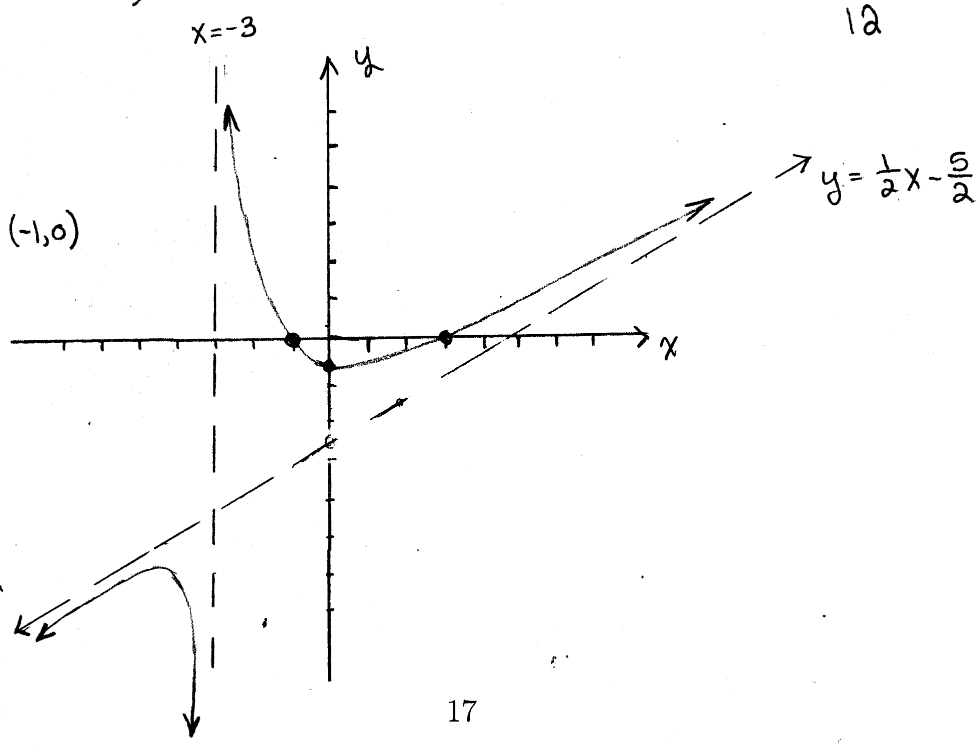
$$y = \frac{(x-3)(x+1)}{2(x+3)} = \frac{1}{2}x - \frac{5}{2} + \frac{12}{2x+6}$$

V.A. $x = -3$

X-INTS: $(3,0)$, $(-1,0)$

Y-INT: $\left(0, -\frac{1}{2}\right)$

PLOT SOME POINTS & USE GRAPHING CALC.

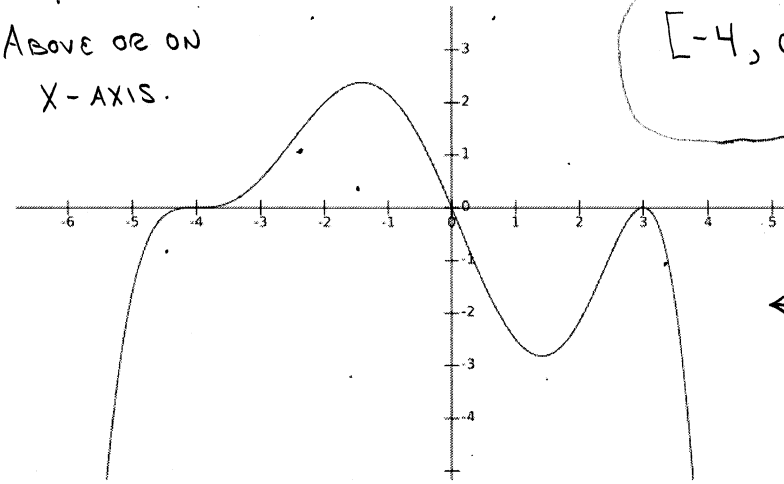


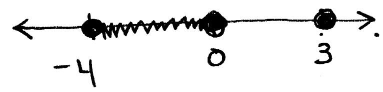
Objective: Solve polynomial inequalities. [5,9,10]

55. Let $p(x)$ be the polynomial whose graph is shown below. Use the graph to solve the inequality $\underbrace{p(x) \geq 0}$.

ABOVE OR ON X-AXIS.

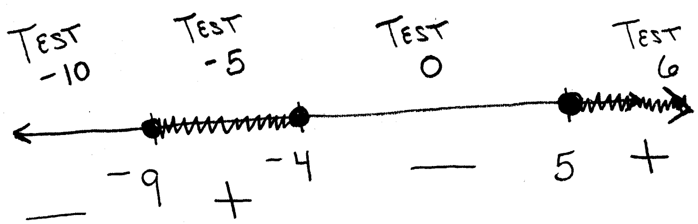
$\boxed{[-4, 0] \cup \{3\}}$





56. Solve and graph the solution set on a number line: $(x + 4)(x + 9)(x - 5) \geq 0$

$x = -4 \quad x = -9 \quad x = 5$



$[-9, -4] \cup [5, \infty)$

57. Solve and graph the solution set on a number line: $x^3 + 5x^2 \leq 4x + 20$

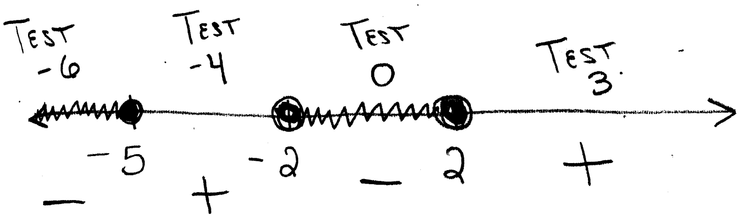
$x^3 + 5x^2 - 4x - 20 \leq 0$

$x^2(x+5) - 4(x+5) \leq 0$

$(x^2 - 4)(x+5) \leq 0$

$(x-2)(x+2)(x+5) \leq 0$

$x = 2 \qquad x = -2 \qquad x = -5$

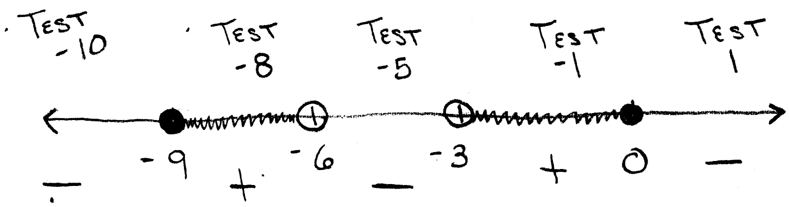


$(-\infty, -5] \cup [-2, 2]$

Objective: Solve rational inequalities. [5,9,10,11]

58. Solve and graph the solution set on a number line: $f(x) = \dfrac{-x^2 - 9x}{x^2 + 9x + 18} \geq 0$

$$\frac{-x(x+9)}{(x+3)(x+6)} \geq 0$$



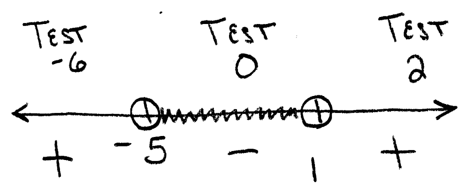
Zeros: $x = 0,\ x = -9$

Restricted Values: $x = -3,\ x = -6$

$\boxed{[-9, 6) \cup (-3, 0]}$

59. Solve and graph the solution set on a number line: $f(x) = \dfrac{x-1}{x+5} < 0$

$$\frac{x-1}{x+5} < 0$$



Zeros: $x = 1$

Restrict: $x = -5$

$\boxed{(-5, 1)}$

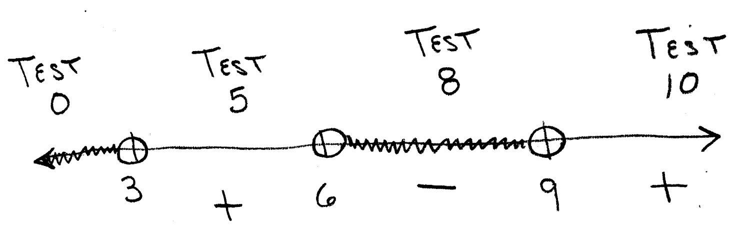
60. Solve and graph the solution set on a number line: $\dfrac{-1}{x-6} < \dfrac{2}{9-x}$

$$-\frac{1}{x-6} - \frac{2}{9-x} < 0$$

Zeros: $x = 3$

Restrict: $x = 6,\ x = 9$

$$\frac{-(9-x) - 2(x-6)}{(x-6)(9-x)} < 0$$



$$\frac{-x+3}{(x-6)(9-x)} < 0$$

$\boxed{(-\infty, 3) \cup (6, 9)}$