Math 109 - Review 4

Name key

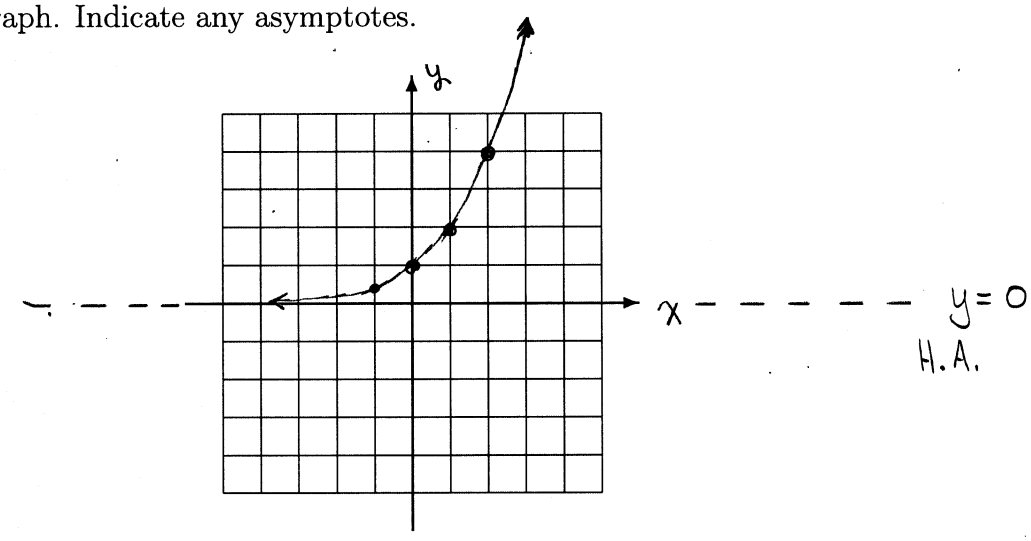
December 8, 2019

These problems may help you review for the final exam. They are coded to match the course objectives from your syllabus. Unless otherwise indicated, you should simplify all answers by reducing fractions, simplifying radicals, and/or rationalizing denominators (as you've done on your ALEKS homework). Label your axes when graphing. **To adequately prepare for the comprehensive final exam, you should also study the earlier review packets, as well as old tests.**

Objective: Evaluate and graph exponential functions. [12]

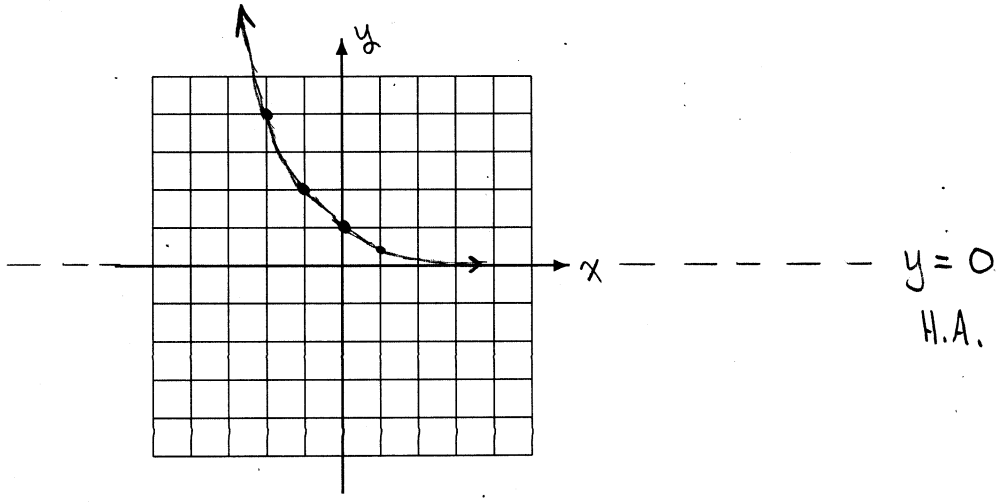
1. Determine four points on the graph of $f(x) = 2^x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

x	$y = 2^x$
0	1
-1	$\frac{1}{2}$
1	2
2	4



2. Determine four points on the graph of $g(x) = \left(\frac{1}{2}\right)^x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

x	$y = \left(\frac{1}{2}\right)^x$
0	1
-1	2
-2	4
1	$\frac{1}{2}$



3. Determine the horizontal asymptote of the graph of $y = e^{x-2} - 3$.

$y = e^x$ & $y = e^{x-2}$ HAVE $y = 0$ AS A H.A.

$y = e^{x-2} - 3$ HAS $y = -3$

4. Determine the horizontal asymptote of the graph of $y = \left(\frac{2}{7}\right)^{x+4} + 9$.

$y = \left(\frac{2}{7}\right)^x$ & $y = \left(\frac{2}{7}\right)^{x+4}$ HAVE $y = 0$ AS A H.A.

$y = \left(\frac{2}{7}\right)^{x+4} + 9$ HAS $y = 9$

5. Determine the y-intercept of the graph of $y = 5^{x+1} + 8$.

$X = 0 \Rightarrow y = 5 + 8 = 13 \rightarrow (0, 13)$

Objective: Convert between logarithmic and exponential notation. [12]

6. Rewrite as an exponential equation: $\log_7 49 = 2$

$7^2 = 49$

7. Rewrite as an exponential equation: $\log_{1/2} 64 = -6$

$\left(\frac{1}{2}\right)^{-6} = 64$

8. Rewrite as a logarithmic equation: $3^6 = 729$

$\log_3 729 = 6$

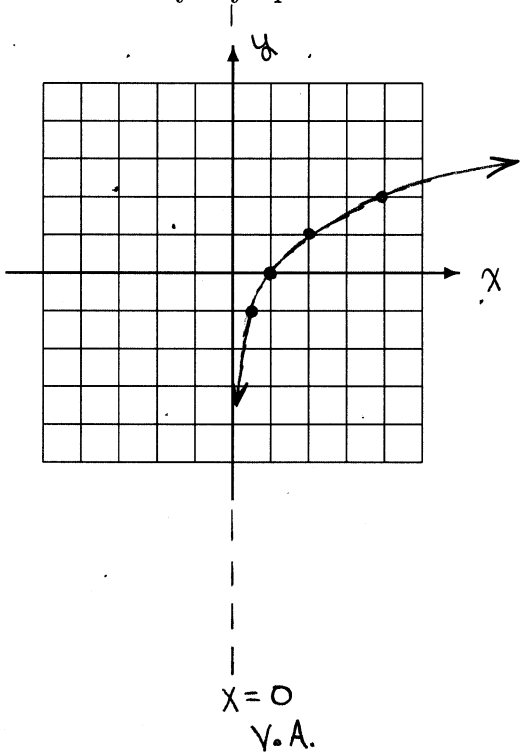
9. Rewrite as a logarithmic equation: $2^{-5} = \frac{1}{32}$

$\log_2 \left(\frac{1}{32}\right) = -5$

Objective: Evaluate and graph logarithmic functions. [12]

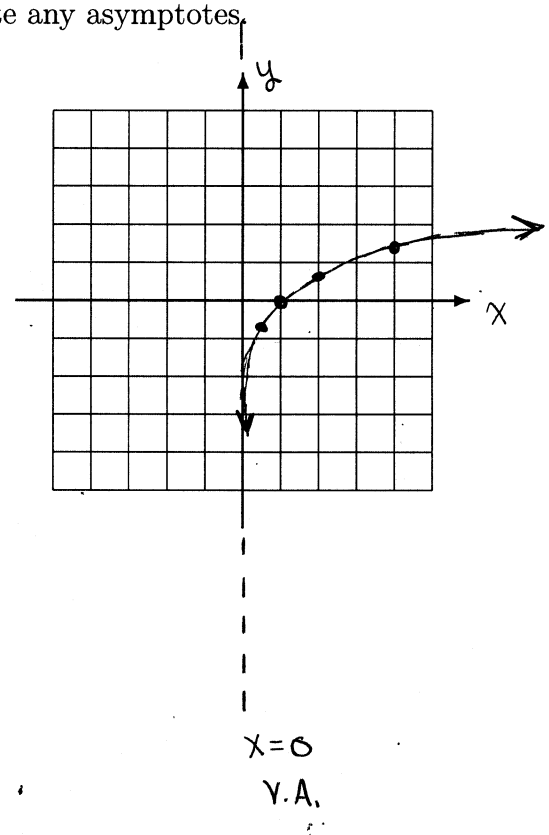
10. Determine four points on the graph of $f(x) = \log_2 x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

x	$y = \log_2 x$
1	0
$\frac{1}{2}$	-1
2	1
4	2



11. Determine four points on the graph of $g(x) = \ln x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

x	$y = \ln x$
1	0
1/2	-0.693
2	0.693
4	1.386



12. Explain how the graph of $y = 3 + \ln(x-4)$ can be obtained from the graph of $y = \ln x$.

START WITH THE GRAPH OF $y = \ln x$,
THEN SHIFT IT RIGHT 4 UNITS
AND UP 3 UNITS.

13. Determine the vertical asymptote of the graph of $y = \ln(x-3)$.

$y = \ln x$ HAS $x = 0$ AS ITS V.A.

$\Rightarrow$ $y = \ln(x-3)$ HAS $\boxed{x = 3}$

14. Determine the vertical asymptote of the graph of $y = 4 + \log_5(x+2)$.

$y = \log_5 x$ HAS $x = 0$ AS ITS V.A.

$\Rightarrow$ $y = 4 + \log_5(x+2)$ HAS $\boxed{x = -2}$

15. Determine the x-intercept of the graph of $y = \log_2(x-7)$.

$\log_2(x-7) = 0$

WHEN $x - 7 = 1$

$\Rightarrow$ $x = 8$

$\therefore$ $\boxed{\text{X-INT IS } (8,0).}$

Objective: Use properties of logarithms to simplify, expand, and condense expressions. [9]

16. Use properties of logarithms to expand: $\ln(xyz)$

$\ln x + \ln y + \ln z$

17. Use properties of logarithms to expand: $\log\left(\frac{u^4}{w^3}\right)$

$4\log u - 3\log w$

18. Use properties of logarithms to expand: $\log_2\left(\frac{a^2b^3}{c^4d^5}\right)$

$2\log_2 a + 3\log_2 b - 4\log_2 c - 5\log_2 d$

19. Write as a single logarithmic expression: $\ln x + \ln 5$ $= \ln 5 + \ln x$

$\ln 5x$

20. Write as a single logarithmic expression: $2\log x + 5\log y - 7\log z$

$\log\left(\frac{x^2y^5}{z^7}\right)$

21. Simplify: $\ln e^8 + \ln e^4$

$= 8\ln e^1 + 4\ln e^1$

$= 8 + 4 = 12$

Objective: Apply the change-of-base formula. [9]

22. Use the change-of-base formula to write $\log_5 6$ in terms of natural logarithms. Then use your calculator to compute the value. Round to the nearest hundredth.

$$\log_5 6 = \frac{\ln 6}{\ln 5} \approx 1.11$$

23. Use the change-of-base formula to write $\log_7 63$ in terms of common (base-10) logarithms. Then use your calculator to compute the value. Round to the nearest hundredth.

$$\log_7 63 = \frac{\log 63}{\log 7} \approx 2.13$$

Objective: Solve exponential or logarithmic equations. [9]

24. Solve for x: $16 = 8^{x-3}$

$$2^4 = (2^3)^{x-3}$$
$$2^4 = 2^{3(x-3)}$$
$$4 = 3(x-3)$$
$$4 = 3x - 9$$
$$13 = 3x \Rightarrow x = \frac{13}{3}$$

25. Solve for x: $\frac{4\log_2(x+12)}{4} = \frac{12}{4}$

$$\log_2(x+12) = 3$$
$$2^3 = x + 12$$
$$8 = x + 12$$
$$x = -4$$

26. Solve for x. Round your answer to the nearest hundredth. $2^{3x} = 15$

$$\ln 2^{3x} = \ln 15$$

$$\frac{3x \ln 2}{3 \ln 2} = \frac{\ln 15}{3 \ln 2}$$

$$x \approx 1.30$$

27. Solve for x. Round your answer to the nearest hundredth. $\frac{4\ln(x+6)}{4} = \frac{-8}{4}$

$$\ln(x+6) = -2$$

$$e^{-2} = x + 6 \rightarrow x = e^{-2} - 6 \approx -5.86$$

28. Solve for x. Round your answer to the nearest hundredth. $\ln(x+3) - \ln 2 = 3$

$$\ln\left(\frac{x+3}{2}\right) = 3$$

$$\frac{x+3}{2} = e^3$$

$$x + 3 = 2e^3 \rightarrow x = 2e^3 - 3 \approx 37.17$$

29. Solve for x. Round your answer to the nearest hundredth. $\log_5(x-7) = 1 + \log_5(x+1)$

$$\log_5(x-7) - \log_5(x+1) = 1$$

$$\log_5\left(\frac{x-7}{x+1}\right) = 1$$

$$\frac{x-7}{x+1} = 5 \rightarrow x - 7 = 5x + 5$$

$$-12 = 4x$$

$$x = -3$$

BUT THIS CAN'T BE A SOLUTION.

NO SOLUTION

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Objective: Solve compound interest problems. [13]

30. Eve deposits \$7500 into an account earning 4.5% compounded monthly. What is the value of the account after 10 years?

$$A = 7500\left(1 + \frac{0.045}{12}\right)^{12 \cdot 10}$$

$$= \boxed{\$\,11,752.45}$$

31. Sal takes a student loan of \$8000 at the rate of 7.9% compounded semiannually. He make no loan payments until he pays the loan in full after 5 years. How much will he owe in 5 years when the loan comes due?

$$A = 8000\left(1 + \frac{0.079}{2}\right)^{2 \cdot 5}$$

$$= \boxed{\$\,11,785.14}$$

32. When she took her new job, Marie received a \$15,000 cash bonus. She invested the money in a bond fund that earns 6.59% compounded quarterly. How long must she wait for the investment to double its value?

$$30000 = 15000\left(1 + \frac{0.0659}{4}\right)^{4t}$$

Solve for t.

$$2 = \left(1 + \frac{0.0659}{4}\right)^{4t}$$

$$\frac{\ln 2}{4\ln\left(1 + \frac{0.0659}{4}\right)} = \frac{4t \ln\left(1 + \frac{0.0659}{4}\right)}{4\ln\left(1 + \frac{0.0659}{4}\right)}$$

$$t = \frac{\ln 2}{4\ln\left(1 + \frac{0.0659}{4}\right)}$$

$$\boxed{t \approx 10.6 \text{ years}}$$

Objective: Solve 2-by-2 linear systems graphically. [14]

33. Solve the system by graphing each equation.

$$x - y = 2$$

$$\frac{1}{2}x + y = 4$$

$x - y = 2$

INTERCEPTS ARE

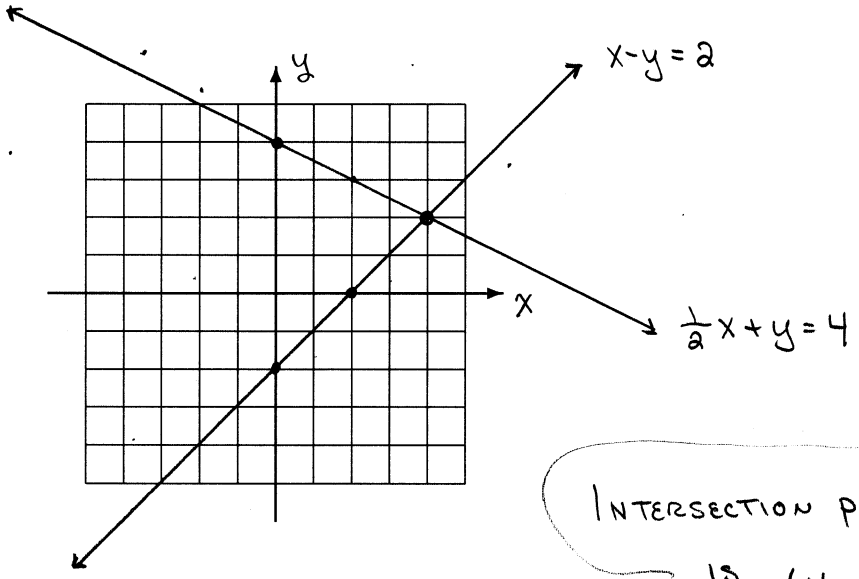
$(2,0)$ & $(0,-2)$

$\frac{1}{2}x + y = 4$

$y = -\frac{1}{2}x + 4$

Y-INT $(0,4)$

SLOPE $-\frac{1}{2}$



INTERSECTION POINT IS $(4,2)$.

34. Solve the system by graphing each equation.

$$2x + 3y = 6$$

$$x = 4 - \frac{3}{2}y$$

$2x + 3y = 6$

INTERCEPTS ARE

$(3,0)$ & $(0,2)$

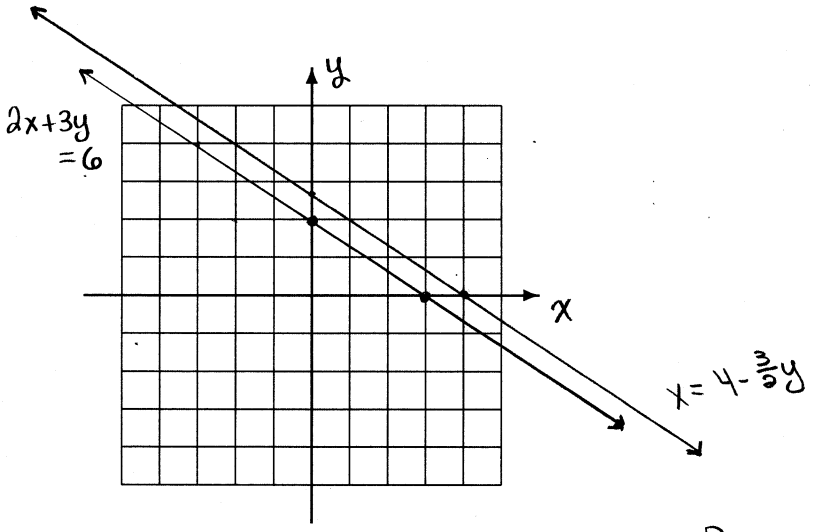
$x = 4 - \frac{3}{2}y$

$2x = 8 - 3y$

$2x + 3y = 8$

INTERCEPTS ARE

$(4,0)$ & $(0, \frac{8}{3})$



DISTINCT, PARALLEL LINES.

NO SOLUTION.

Objective: Solve 2-by-2 and 3-by-3 linear systems. [14]

35. Use any method to solve.

$-3x + 2y = 13$

$x = 3y - 9$

$-3(3y-9) + 2y = 13$

$-9y + 27 + 2y = 13$

$-7y = -14$

$y = 2$

$x = 3(2) - 9 = -3$

$(-3, 2)$

36. Use any method to solve.

$-3x + y = 17$

$-(7x + y = -23)$

$-10x = 40$

$x = -4$

$(-4, 5)$

$-3(-4) + y = 17$

$y = 5$

37. Use any method to solve.

$3(-5x + 2y = 8)$ $\quad -15x + 6y = 24$

$2(-8x - 3y = 19)$ $\quad -16x - 6y = 38$

$-31x = 62$

$x = -2$

$(-2, -1)$

$-5(-2) + 2y = 8$

$2y = -2$

$y = -1$

38. Use any method to solve.

$$6\left(\frac{1}{3}x+\frac{1}{2}y=10\right)$$
$$\frac{1}{5}x-3y=-\frac{3}{5}$$

$2x + 3y = 60$

$\frac{1}{5}x - 3y = -\frac{3}{5}$

$\frac{2\frac{1}{5}x}{2\frac{1}{5}} = \frac{59\frac{2}{5}}{2\frac{1}{5}}$

$x = 27$

$\frac{1}{3}(27) + \frac{1}{2}(y) = 10$

$9 + \frac{1}{2}y = 10 \Rightarrow y = 2$

(27, 2)

39. Use any method to solve.

$$-6\left(-1.4x+0.6y=8\right)$$
$$0.6x+3.6y=-15$$

$8.4x - 3.6y = -48$

$0.6x + 3.6y = -15$

$9x = -63$

$x = -7$

$0.6(-7) + 3.6y = -15$

$-4.2 + 3.6y = -15$

$3.6y = -10.8$

$y = -3$

(-7, -3)

40. Use any method to solve.

$$\left\{\begin{array}{l} 2x+y+z=-3 \\ -2x+2y+z=4 \\ 2\left(-x+y+2z=5\right) \end{array}\right\}$$

ADD THESE.

ADD THESE.

$3y + 2z = 1$

$3y + 5z = 7$

$3y + 2z = 1$

$-3y - 5z = -7$

$-3z = -6$

$z = 2$

$3y + 2(2) = 1$

$y = -1$

$2x + (-1) + 2 = -3$

$2x = -4$

$x = -2$

(-2, -1, 2)

41. Use any method to solve.

$3\left(-4x+y=-4\right)$

$12x-3y=-12$

$-12x+3y=-12$

$12x-3y=-12$

$0=-24$

No solution.

42. Use any method to solve.

$$\begin{cases} 2x-y+3z=-5 \\ -x+y-z=1 \\ 4x-3y+5z=-7 \end{cases}$$

$2x-y+3z=-5$

$-2x+2y-2z=2$

$y+z=-3$

$4x-2y+6z=-10$

$-4x+3y-5z=7$

$y+z=-3$

Infinitely many solutions.

$z=-3-y$

$-x+y-z=1$

$-x+y-(-3-y)=1$

$-x+3+2y=1$

$-x+2y=-2$

$x=2+2y$

y can be any number.

Then

$x=2+2y$

$z=-3-y$

Objective: Set up and solve a system of linear equations in an application. [14]

43. One month Jina rented 8 movies and 3 video games for a total of \$27. The next month she rented 2 movies and 5 video games for a total of \$28. Find the rental cost for each movie and each video game. Set up and solve the system of equations.

Let m = # of movies
v = # of video games

$$\left.\begin{aligned} 8m + 3v &= 27 \\ 2m + 5v &= 28 \end{aligned}\right\}$$

$$\begin{aligned} 8m + 3v &= 27 \\ -8m - 20v &= -112 \\ \hline -17v &= -85 \\ v &= 5 \end{aligned}$$

$$\begin{aligned} 2m + 5(5) &= 28 \\ 2m &= 3 \\ m &= 1.5 \end{aligned}$$

$v = \$5$
$m = \$1.50$

44. Lena, Michael, and Chris have a total of \$85 in their wallets. Chris has \$5 less than Lena. Michael has 2 times what Chris has. How much does each have in his/her wallet? Set up and solve the system of equations.

l = amount for Lena.
m = amount for Michael
c = amount for Chris

$$l + m + c = 85$$
$$l = c + 5$$
$$m = 2c$$

$$(c+5) + 2c + c = 85$$
$$4c + 5 = 85$$
$$4c = 80$$
$$c = 20$$

$m = 40$
$l = 25$

$c = \$20$
$m = \$40$
$l = \$25$

45. Rachel, Chris, and Ali sent a total of 125 text messages over their cell phones during the weekend. Rachel sent 7 more messages than Chris. Ali sent 4 times as many messages as Rachel. How many messages did they each send? Set up and solve the system of equations.

r = Rachel # of texts
c = Chris # of texts
a = Ali # of texts

$$r + c + a = 125$$
$$r = c + 7$$
$$a = 4r$$

$$r + (r-7) + 4r = 125$$
$$6r - 7 = 125$$
$$6r = 132$$
$$r = 22$$

$c = 15$
$a = 88$

$r = 22$
$c = 15$
$a = 88$