Math 109 - Test 3A

November 14, 2019

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary. Label your axes when graphing. You may get partial credit on multiple choice problems if you show correct work or explanations.

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1. (4 points [6]) The graph of $y = \sqrt{x}$ is reflected about one of the coordinate axes. What is the new equation if the reflection is about

   (a) the x-axis? $y = -\sqrt{x}$

   (b) the y-axis? $y = \sqrt{-x}$

2. (6 points [6]) Describe the sequence of transformations (in order) that transform the graph of $y = x^3$ to that of $y = -(x+1)^3 + 5$.

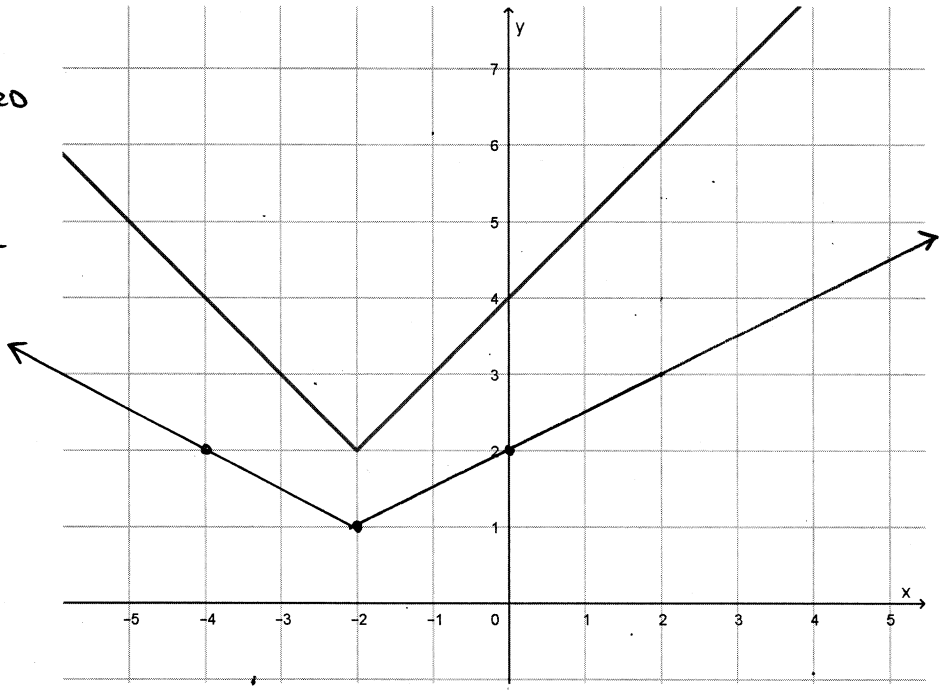
   ① Shift left 1 unit, ② Reflect about x-axis,

   ③ Shift up 5 units

3. (4 points [6]) The graph of $y = f(x)$ is shown below. On the same grid, carefully sketch the graph of $y = \frac{1}{2}f(x)$.

Every y-coord is one-half its original value...

Vertical Compression.



4. (6 points [6]) The functions f and g are defined in the table shown below. Use the data from the table to evaluate each of the following.

x	1	2	3	4	5
$f(x)$	-1	3	2	5	0
$g(x)$	7	0	4	-7	4

(a) $(fg)(3)$ $f(3)\,g(3) = (2)(4) =$ 8

(b) $\left(\frac{g}{f}\right)(5)$ $\frac{g(5)}{f(5)} = \frac{4}{0}$ NOT DEFINED

(c) $(f+f)(4)$ $f(4) + f(4) = (5) + (5) =$ 10

(d) $(f \circ g)(3)$ $f(g(3)) = f(4) =$ 5

5. (4 points [6]) Let $f(x) = x^2 + x$ and $g(x) = x + 4$. Find and simplify the formula for

(a) $(g \circ f)(x)$. $g(f(x)) =$ $x^2 + x + 4$

(b) $(f \circ g)(x)$. $(x+4)^2 + (x+4) = x^2 + 8x + 16 + x + 4$

$f(g(x)) =$

$x^2 + 9x + 20$

6. (3 points [6]) Find two functions f and g so that $(f \circ g)(x) = \sqrt[3]{3x^2 + 7}$.

$g(x) = 3x^2 + 7$

$f(x) = \sqrt[3]{x}$

7. (4 points [5,7,8]) Write the quadratic function $f(x) = x^2 - 8x + 12$ in vertex form.

Complete square

$x^2 - 8x + 16 - 4$

$f(x) = (x-4)^2 - 4$

-or-

Use Formula

Vertex at $-\frac{b}{2a} = 4$

$f(4) = 16 - 32 + 12 = -4$

$f(x) = (x-4)^2 - 4$

8. (6 points [5,7,8]) The graph of $f(x) = 17(x+9)^2 - 12$ is a parabola.

(a) Does the parabola open upward or downward? How do you know?

Upward, $a = 17 > 0$

(b) Find the vertex of the parabola.

$(-9, -12)$

(c) Write an equation for the parabola's symmetry axis.

$x = -9$

(d) Based on your answers for parts (a) and (b), does the equation $f(x) = 0$ have zero, one, or two real solutions? Explain.

Two real solutions. Parabola opens up & vertex is below the x-axis.

9. (6 points [9]) Use synthetic division to find the quotient and remainder.

$(2x^3 + 3x^2 + 1) \div (x + 3)$

-3	2	3	0	1
		-6	9	-27
	2	-3	9	-26

Quotient: $2x^2 - 3x + 9$

Remainder: -26

10. (9 points [9]) Use long division to find the quotient and remainder.

$$(2x^3 + 7x^2 + 7x + 1) \div (x^2 + 2x)$$

$$\begin{array}{r} 2x+3 \\ x^2+2x \overline{)\,2x^3+7x^2+7x+1} \\ -(2x^3+4x^2) \\ \hline 3x^2+7x+1 \\ -(3x^2+6x) \\ \hline x+1 \end{array}$$

Quotient: $2x+3$

Remainder: $x+1$

$$2x+3+\frac{x+1}{x^2+2x}$$

11. (5 points [9]) Let $f(x) = x^3 - 4x^2 + 3x + 8$. Evaluate $f(-1)$. Based on the value of $f(-1)$, determine whether $(x+1)$ is a factor of f.

$$f(-1) = (-1)^3 - 4(-1)^2 + 3(-1) + 8$$

$$= -1 - 4 - 3 + 8 = 0$$

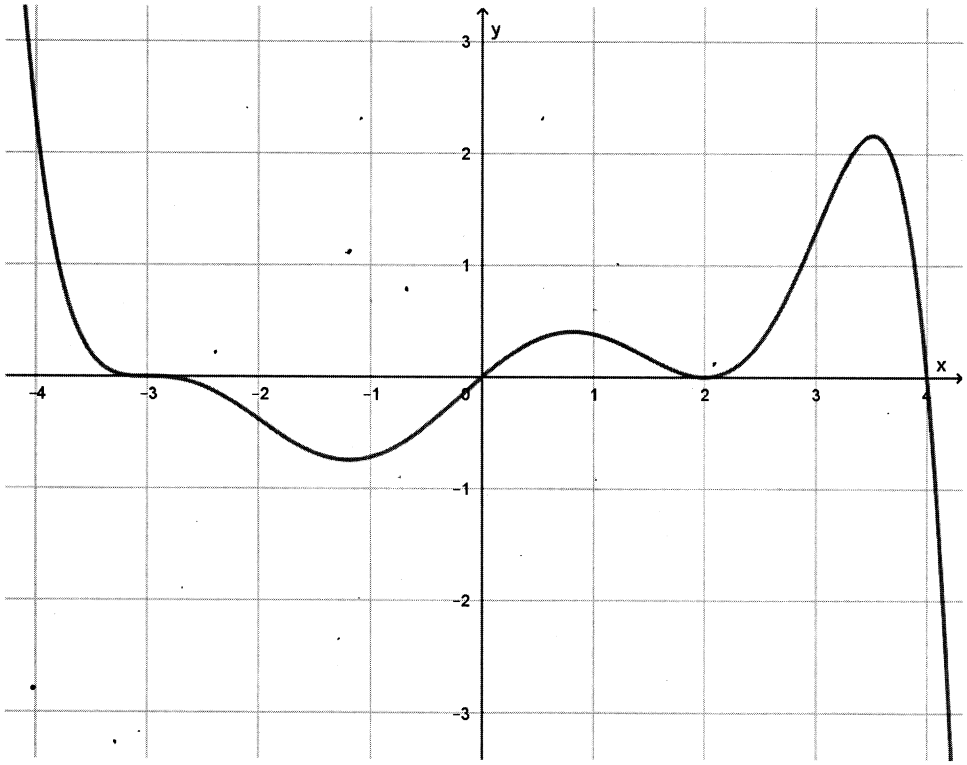
$f(-1) = 0 \Rightarrow x+1$ is a factor of f

12. (5 points [10,11]) Determine the vertical asymptote(s) of the graph of $R(x) = \dfrac{2x-2}{(x-6)(x+1)}$.

$$R(x) = \frac{2(x-1)}{(x-6)(x+1)}$$

$x = 6$ is the only V.A.

13. (12 points [10]) The graph of a polynomial is shown below.



(a) Is the degree even or odd?

Odd

(b) Is the leading coefficient positive or negative?

Negative

(c) Which zeros have multiplicity one?

$x = 0, \quad x = 4$

(d) Which zeros have even multiplicity?

$x = 2$

(e) Which zeros have odd multiplicity greater than 1?

$x = -3$

(f) Write the factored form of a polynomial whose graph has the same general shape.

$p(x) = -1(x+3)^3(x)(x-2)^2(x-4)$

14. (10 points [5,9,10]) Solve the inequality and write your solution in interval notation.

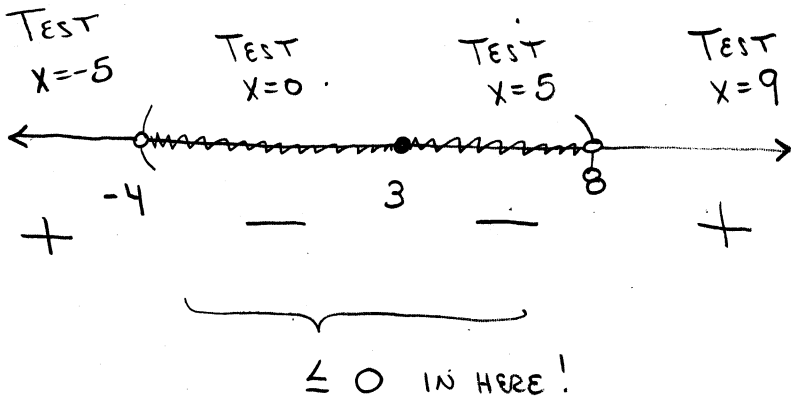
$$\frac{(x-3)^2}{(x+4)(x-8)} \leq 0$$

INCLUDE ZEROS

ZEROS OF NUM : $x=3$

ZEROS OF DENOM :

$x=-4, x=8$



≤ 0 IN HERE!

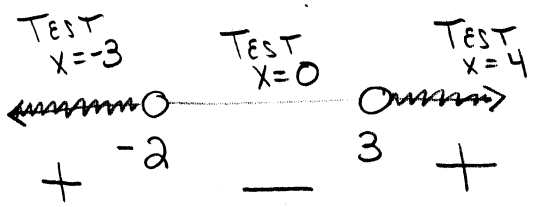
$(-4, 8)$

15. (6 points [5,9,10]) Solve the inequality and write your solution in interval notation.

$$x^2 - x > 6$$

$x^2 - x - 6 > 0$

$(x-3)(x+2) > 0$



$(-\infty, -2) \cup (3, \infty)$

16. (2 points [6]) The graph of $y = 13 + 20(x-35)^2$ is reflected about the x-axis. Which one of the following is an equation for the new transformed graph?

(a) $y = -13 + 20(x-35)^2$

(b) $y = 13 - 20(x-35)^2$

(c) $y = -13 + 20(x+35)^2$

(d) $y = -13 - 20(x-35)^2$

$y = f(x) \longrightarrow y = -f(x)$

17. (2 points [9]) Suppose $f(x)$ is a polynomial function and $f(2) = 0$. Which one of the following is true?

(a) $(x+2)$ is a factor of f.

(b) $(x-2)$ is a factor of f.

(c) When $f(x)$ is divided by x, the remainder is 2.

(d) When $f(x)$ is divided by $x-2$, the remainder is 2.

18. (2 points [9,10]) The graph of a polynomial function rises to the left and falls to the right. Which one of the following must be true?

(a) The degree is odd and the leading coefficient is positive.

(b) The degree is odd and the leading coefficient is negative.

(c) The degree is even and the leading coefficient is positive.

(d) The degree is even and the leading coefficient is negative.

19. (2 points [10,11]) Which one of the following is true of the graph of $g(x) = \dfrac{3x^3 + 8x^2 - 1}{(x-2)^2}$?

(a) The graph has a horizontal asymptote.

(b) The graph has two vertical asymptotes.

(c) The graph has a slant asymptote.

(d) The graph has no asymptotes.

20. (2 points [10,11]) Which of the following is the horizontal asymptote of the graph of $y = \dfrac{-412x + 13}{(x+1)(x-1)}$?

(a) $y = 0$

(b) $y = -412$

(c) $y = 13$

(d) None of the above