Math 109 - Final Exam A

December 12, 2019

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. Label your axes when graphing.

1. (4 points [3]) Solve for w: $-3(w+5)+5w = 3w+10-(w+15)$

$$-3w - 15 + 5w = 3w + 10 - w - 15$$
$$2w - 15 = 2w - 5$$
$$-15 = -5 \quad \text{CONTRADICTION.}$$

No soln.

2. (6 points [3]) Solve for y. Write your solution set in interval notation, and graph it on a number line.

$$2(y+2)-3 < y+7 \quad \text{and} \quad 7-2y \le 1$$

$2y + 4 - 3 < y + 7$

$2y + 1 < y + 7$

$y < 6$

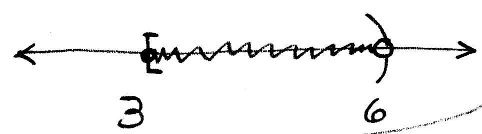
- AND -

$7 - 2y \le 1$

$-2y \le -6$

$y \ge 3$

$[3, 6)$



3. (5 points [7]) Solve for x. Write your answer(s) in decimal form, rounded to the nearest hundredth.

$$4x^2 - 4x - 1 = 0$$

QUAD. FORMULA. $a = 4$, $b = -4$, $c = -1$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{4 \pm \sqrt{32}}{8} = \frac{4 \pm 4\sqrt{2}}{8} = \frac{1 \pm \sqrt{2}}{2}$$

$$x = \frac{1+\sqrt{2}}{2} \approx 1.21$$

$$x = \frac{1-\sqrt{2}}{2} \approx -0.21$$

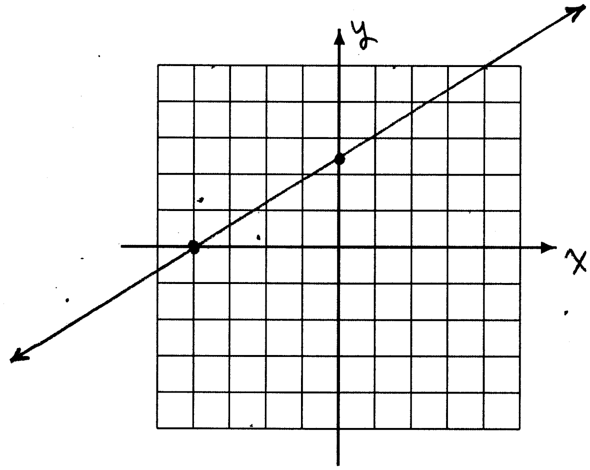
4. (6 points [2]) A line is described by the equation $-\frac{5}{4}x + 2y = 5$. Find the x- and y-intercepts of the line. Then plot your intercepts and sketch the line.

X-INT...

$y = 0 \Rightarrow -\frac{5}{4}x = 5$

$\Rightarrow x = -4$

X-INT : $(-4, 0)$



Y-INT...

$x = 0 \Rightarrow 2y = 5$

$\Rightarrow y = \frac{5}{2}$

Y-INT : $(0, \frac{5}{2})$

5. (6 points [6]) Let $g(x) = x^2 + x$. Expand and simplify the difference quotient $\frac{g(x+h) - g(x)}{h}$.

$$\frac{[(x+h)^2 + (x+h)] - [x^2 + x]}{h} = \frac{(x^2 + 2xh + h^2 + x + h) - (x^2 + x)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h} = \frac{2xh + h^2 + h}{h}$$

$$= \frac{\cancel{h}(2x + h + 1)}{\cancel{h}} = \boxed{2x + h + 1}$$

6. (3 points [1]) Determine the domain of the function $f(x) = \frac{2x}{(2x+1)(x-7)}$.

ZERO DENOM WHEN

$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

OR

$x - 7 = 0 \Rightarrow x = 7$

DOMAIN = ALL REAL #'S EXCEPT $x = -\frac{1}{2}$ & $x = 7$

7. (5 points [2]) The line L passes through the point $(-4, -2)$ and is perpendicular to the line given by $y = -2x + 1$. Find an equation for the line L. Write your final answer in standard form $(Ax + By = C)$.

Slope of L is $\frac{1}{2}$
(opp. recip of -2)

Point: $(-4, -2)$

$y + 2 = \frac{1}{2}(x + 4)$

$y + 2 = \frac{1}{2}x + 2$

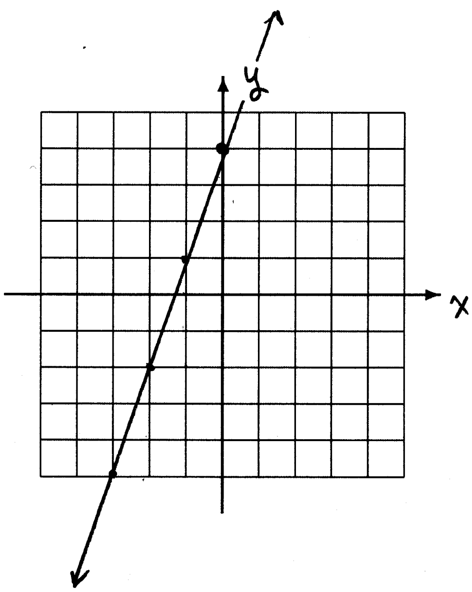
$\boxed{-\frac{1}{2}x + y = 0}$

8. (5 points [2]) Determine the slope and y-intercept of the line described by $3x - y = -4$. Then graph the line.

$3x - y = -4$

$y = 3x + 4$

Slope $= 3$
Y-int is $(0, 4)$



9. (4 points [9]) Use synthetic division and the remainder theorem to evaluate $f(2)$ if $f(x) = 2x^2 + 3x + 1$.

2	2	3	1
+		4	14
	2	7	15

$\boxed{f(2) = 15}$

10. (4 points [6,8]) The graph of $f(x) = (x+3)^2 - 5$ is a parabola.

(a) Explain how the graph of f can be obtained from the graph of $y = x^2$.

SHIFT 3 UNITS LEFT AND 5 UNITS DOWN

(b) Determine the vertex and an equation for the axis of symmetry of the graph of f.

VERTEX $(-3,-5)$ SYMMETRY AXIS : $X = -3$

11. (12 points [9,10]) Consider the polynomial $f(x) = -2x(x-2)^3(x+1)^2$.

(a) Determine the degree of f.

$1+3+2 = \boxed{6}$

(b) State the zeros of f and their corresponding multiplicities.

$X = 0$, MULT 1 $X = -1$, MULT 2

$X = 2$, MULT 3

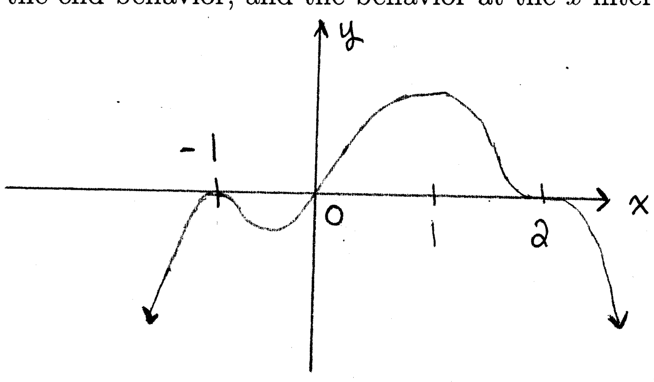
(c) Describe the end behavior of the graph of f.

NEG. LEADING COEFF & DEG 6 $\Longrightarrow$ DOWN LEFT / DOWN RIGHT

(d) Determine the y-intercept.

$X = 0 \Rightarrow y = f(0) = 0$ (0,0)

(e) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the y-intercept, the end behavior, and the behavior at the x-intercepts.

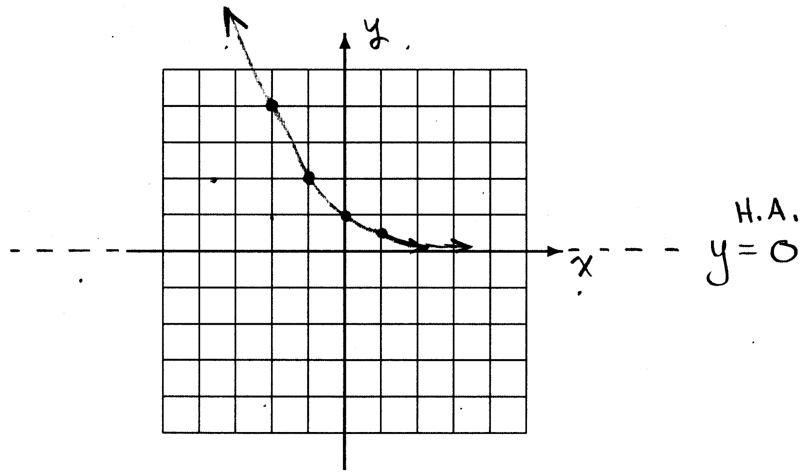


(f) Use your graph to solve $f(x) > 0$. Write your solution in interval notation.

$f(x) > 0$ ON $(0,2)$

12. (5 points [12]) Determine four points on the graph of $g(x) = \left(\frac{1}{2}\right)^x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

x	$y = \left(\frac{1}{2}\right)^x$
0	1
-1	2
-2	4
1	1/2



13. (2 points [12]) Rewrite as an exponential equation: $\log_3\left(\frac{1}{9}\right) = -2$

$$\boxed{3^{-2} = \frac{1}{9}}$$

14. (4 points [12]) Explain how the graph of $f(x) = 3 + \ln(x-4)$ can be obtained from the graph of $y = \ln x$. Then find an equation for the vertical asymptote of the graph of f.

SHIFT THE GRAPH OF $y = \ln x$ 4 UNITS RIGHT & 3 UNITS UP → THIS PUTS THE V.A. FOR THE GRAPH OF f AT $\boxed{x = 4}$

15. (3 points [9]) Use properties of logarithms to completely expand: $\log\left(\frac{x^3}{y^2}\right)$

$$\log x^3 - \log y^2$$

$$= \boxed{3\log x - 2\log y}$$

16. (3 points [9]) Use the change-of-base formula to write $\log_7 91$ in terms of natural logarithms. Then use your calculator to compute the value. Round to the nearest hundredth.

$$\log_7 91 = \boxed{\frac{\ln 91}{\ln 7} \approx 2.32}$$

17. (3 points [9]) Solve for x: $2^{4x+1} = 32$

$$2^{4x+1} = 2^5$$

$$4x + 1 = 5$$

$$4x = 4 \implies \boxed{x = 1}$$

18. (5 points [13]) When she took her new job, Marie received a \$15,000 cash bonus. She deposited the money into an IRA that earns 8.5% compounded annually. How long must she wait for her account to reach \$100,000? (The compound interest formula is $A = P(1 + \frac{r}{n})^{nt}$.)

$$100000 = 15000\left(1 + \frac{0.085}{1}\right)^{(1 \cdot t)} = 15000(1.085)^t$$

$$\frac{100000}{15000} = \frac{20}{3} = (1.085)^t$$

$$\ln \frac{20}{3} = t \ln 1.085 \implies \boxed{t = \frac{\ln \frac{20}{3}}{\ln 1.085} \approx 23.25 \text{ years}}$$

19. (8 points [14]) One month Jina rented 6 movies and 4 video games for a total of \$30. The next month she rented 3 movies and 6 video games for a total of \$33. Set up the system of linear equations described by the problem situation. Then solve your system to determine the cost of each movie and video game rental.

m = COST OF MOVIE

v = COST OF GAME

$$6m + 4v = 30$$
$$3m + 6v = 33$$

$$6m + 4v = 30$$
$$-(6m + 12v = 66)$$
$$-8v = -36$$
$$v = 4.5$$

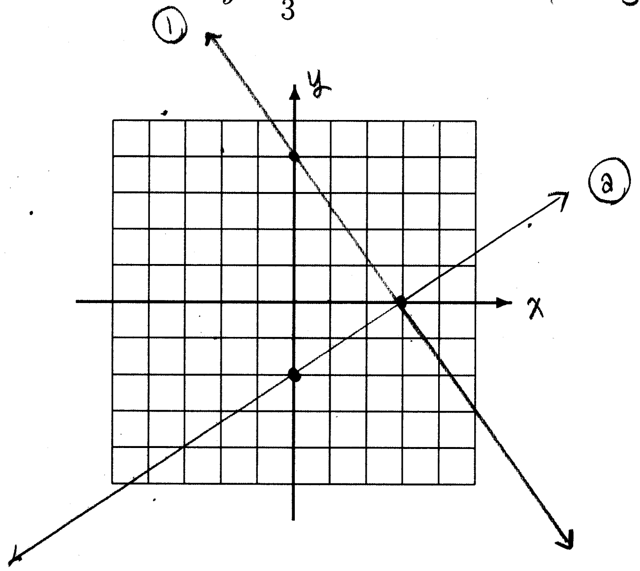
$$6m + 4(4.5) = 30$$
$$6m + 18 = 30$$
$$m = 2$$

$$\boxed{m = \$2, \quad v = \$4.50}$$

20. (7 points [14]) Solve the system by graphing each equation.

① $4x + 3y = 12$ ← INTERCEPTS ARE (3,0) & (0,4)

② $y = \frac{2}{3}x - 2$ ← SLOPE $\frac{2}{3}$, Y-INT (0,-2)



THE LINES INTERSECT AT (3,0).

$\boxed{\text{SOLUTION IS } (3,0).}$