Math 109 - Final Exam B

December 12, 2019

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. Label your axes when graphing.

1. (4 points [3]) Solve for y: $-5(y+4)+8y=4(y-10)-y+20$

$$-5y-20+8y=4y-40-y+20$$

$$3y-20=3y-20.$$

IDENTITY ... EVERY NUMBER IS A SOLN

2. (6 points [3]) Solve for w. Write your solution set in interval notation, and graph it on a number line.

$$3(w+1)-5<w+8 \quad \text{and} \quad 10-3w\leq 1$$

$$3w+3-5<w+8 \qquad 10-3w\leq 1$$

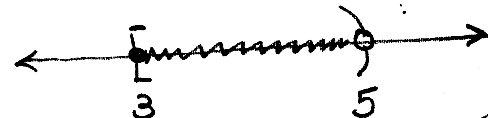
$$3w-2<w+8 \qquad -3w\leq -9$$

$$2w<10 \qquad w\geq 3$$

-AND-

$$w<5$$

$[3,5)$



3. (5 points [7]) Solve for x. Write your answer(s) in decimal form, rounded to the nearest hundredth.

$$2x^2-3x-1=0$$

QUAD FORMULA $a=2$, $b=-3$, $c=-1$

$$x=\frac{3\pm\sqrt{(-3)^2-4(2)(-1)}}{2(2)}=\frac{3\pm\sqrt{17}}{4}$$

$$x=\frac{3+\sqrt{17}}{4}\approx 1.78$$

$$x=\frac{3-\sqrt{17}}{4}\approx -0.28$$

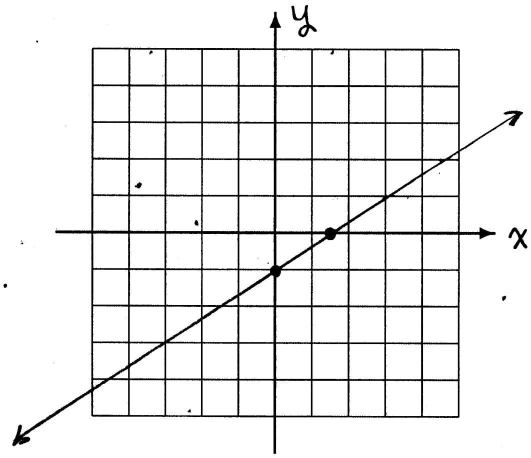
4. (6 points [2]) A line is described by the equation $\frac{8}{3}x - 4y = 4$. Find the x- and y-intercepts of the line. Then plot your intercepts and sketch the line.

X-INT...

$y = 0 \Rightarrow \frac{8}{3}x = 4$

$x = \frac{12}{8} = \frac{3}{2}$

X-INT : $\left(\frac{3}{2}, 0\right)$



Y-INT...

$x = 0 \Rightarrow -4y = 4$

$y = -1$

Y-INT : $(0, -1)$

5. (6 points [6]) Let $g(x) = x^2 + x$. Expand and simplify the difference quotient $\dfrac{g(x+h) - g(x)}{h}$.

$$\frac{\left[(x+h)^2 + (x+h)\right] - \left[x^2 + x\right]}{h} = \frac{(x^2 + 2xh + h^2 + x + h) - (x^2 + x)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h} = \frac{2xh + h^2 + h}{h}$$

$$= \frac{\cancel{h}(2x + h + 1)}{\cancel{h}} = \boxed{2x + h + 1}$$

6. (3 points [1]) Determine the domain of the function $f(x) = \dfrac{2(x-1)}{(2x+5)(x-3)}$.

DENOM IS ZERO WHEN

$2x + 5 = 0 \Rightarrow x = -\frac{5}{2}$

OR

$x - 3 = 0 \Rightarrow x = 3$

DOMAIN = ALL REAL #'S EXCEPT $x = -\frac{5}{2}$ AND $x = 3$.

7. (5 points [2]) The line L passes through the point $(6, 4)$ and is perpendicular to the line given by $y = -2x + 1$. Find an equation for the line L. Write your final answer in standard form $(Ax + By = C)$.

Slope of L is $\frac{1}{2}$
(opp. recip of -2)

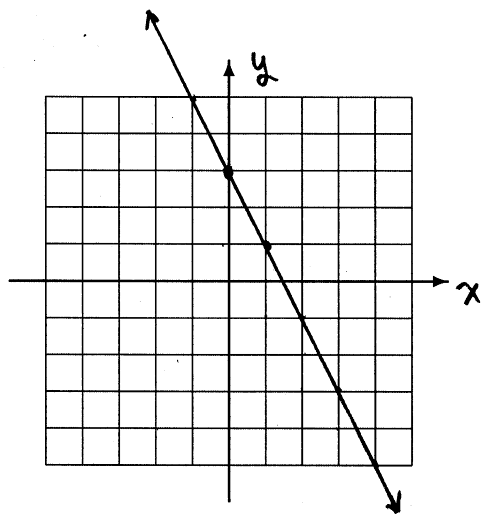
Point: $(6,4)$

$$y - 4 = \frac{1}{2}(x - 6)$$
$$y - 4 = \frac{1}{2}x - 3$$
$$-\frac{1}{2}x + y = 1$$

8. (5 points [2]) Determine the slope and y-intercept of the line described by $2x + y = 3$. Then graph the line.

$2x + y = 3$
$\Downarrow$
$y = -2x + 3$

Slope $= -2$
Y-int $= (0, 3)$



9. (4 points [9]) Use synthetic division and the remainder theorem to evaluate $f(3)$ if $f(x) = 2x^2 - 4x + 5$.

$$\begin{array}{r|rrr} 3 & 2 & -4 & 5 \\ + & & 6 & 6 \\ \hline & 2 & 2 & 11 \end{array}$$

$f(3) = 11$

10. (4 points [6,8]) The graph of $f(x) = (x-4)^2 + 3$ is a parabola.

(a) Explain how the graph of f can be obtained from the graph of $y = x^2$.

SHIFT 4 UNITS RIGHT AND 3 UNITS UP

(b) Determine the vertex and an equation for the axis of symmetry of the graph of f.

VERTEX: $(4,3)$ · SYMMETRY AXIS: $X = 4$

11. (12 points [9,10]) Consider the polynomial $f(x) = 3x(x-3)^2(x+2)^3$.

(a) Determine the degree of f.

$1 + 2 + 3 = \boxed{6}$

(b) State the zeros of f and their corresponding multiplicities.

$X = 0$, MULT 1 $X = -2$, MULT 3

$X = 3$, MULT 2

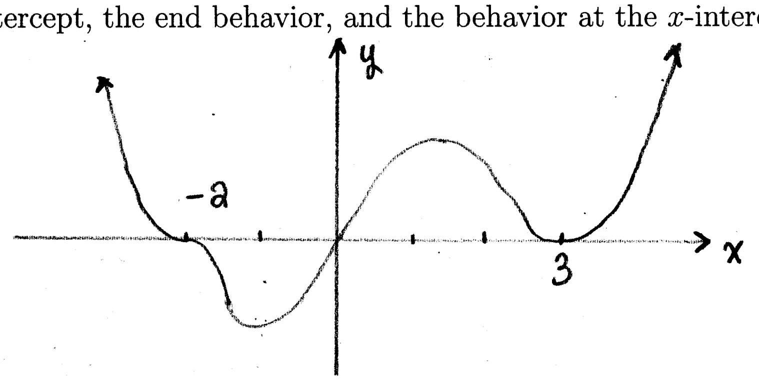
(c) Describe the end behavior of the graph of f.

POS LEADING COEFF & DEG 6 $\Rightarrow$ UP LEFT / UP RIGHT

(d) Determine the y-intercept.

$X = 0 \Rightarrow y = f(0) = 0$ (0,0)

(e) Roughly sketch the graph of f. Be sure that your graph correctly illustrates the y-intercept, the end behavior, and the behavior at the x-intercepts.

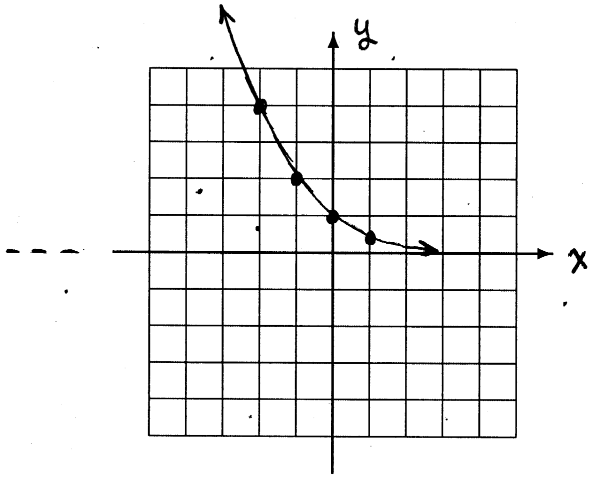


(f) Use your graph to solve $f(x) < 0$. Write your solution in interval notation.

$f(x) < 0$ ON $(-2, 0)$

12. (5 points [12]) Determine four points on the graph of $g(x) = \left(\frac{1}{2}\right)^x$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.

X	$y = \left(\frac{1}{2}\right)^x$
0	1
1	1/2
-1	2
-2	4



H.A. $y = 0$

13. (2 points [12]) Rewrite as an exponential equation: $\log_5 125 = 3$

$\boxed{5^3 = 125}$

14. (4 points [12]) Explain how the graph of $f(x) = 4 + \ln(x - 3)$ can be obtained from the graph of $y = \ln x$. Then find an equation for the vertical asymptote of the graph of f.

SHIFT THE GRAPH OF $y = \ln x$ 3 UNITS RIGHT AND 4 UNITS UP

THIS PUTS THE V.A. OF THE GRAPH OF f AT $\boxed{X = 3}$

15. (3 points [9]) Use properties of logarithms to completely expand: $\log\left(\frac{ab}{c}\right)$

$\log(ab) - \log c$

$\boxed{\log a + \log b - \log c}$

16. (3 points [9]) Use the change-of-base formula to write $\log_3 99$ in terms of natural logarithms. Then use your calculator to compute the value. Round to the nearest hundredth.

$$\log_3 99 = \boxed{\frac{\ln 99}{\ln 3} \approx 4.18}$$

17. (3 points [9]) Solve for x: $2^{2x+1} = 16$

$$2^{2x+1} = 2^4 \Rightarrow 2x+1 = 4$$

$$2x = 3 \quad \boxed{x = \frac{3}{2}}$$

18. (5 points [13]) When she took her new job, Marie received a \$18,000 cash bonus. She deposited the money into an IRA that earns 7.9% compounded annually. How long must she wait for her account to reach \$120,000? (The compound interest formula is $A = P(1 + \frac{r}{n})^{nt}$.)

$$120000 = 18000\left(1 + \frac{0.079}{1}\right)^{(1 \cdot t)} = 18000(1.079)^t$$

$$\frac{120000}{18000} = \frac{20}{3} = 1.079^t$$

$$\ln \frac{20}{3} = t \ln 1.079 \Rightarrow \boxed{t = \frac{\ln \frac{20}{3}}{\ln 1.079} \approx 24.95 \text{ years}}$$

19. (8 points [14]) One month Jina rented 6 movies and 4 video games for a total of \$41. The next month she rented 4 movies and 2 video games for a total of \$24. Set up the system of linear equations described by the problem situation. Then solve your system to determine the cost of each movie and video game rental.

m = COST OF MOVIE

v = COST OF GAME

$$6m + 4v = 41$$
$$4m + 2v = 24$$

$$6m + 4v = 41$$
$$-(8m + 4v = 48)$$
$$-2m = -7$$
$$m = 3.5$$

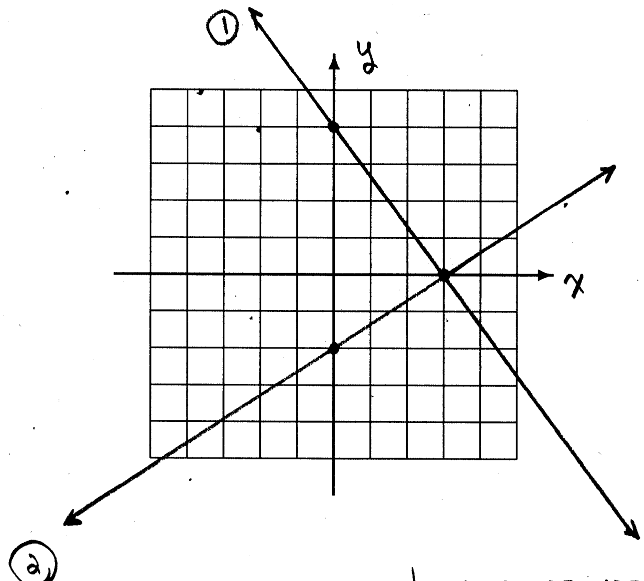
$$6(3.5) + 4v = 41$$
$$21 + 4v = 41$$
$$4v = 20 \quad v = 5$$

$$\boxed{\begin{aligned} v &= \$5 \\ m &= \$3.50 \end{aligned}}$$

20. (7 points [14]) Solve the system by graphing each equation.

① $4x + 3y = 12$ INTERCEPTS ARE (3,0) & (0,4)

② $y = \frac{2}{3}x - 2$ SLOPE IS $\frac{2}{3}$, y-INT IS (0,-2)



INTERSECT AT (3,0)

$\boxed{\text{SOLUTION IS } (3,0).}$