Math 109 - Review 2
March 8, 2020

Name key
Score

These problems may help you review for Test 2. They are coded to match the course objectives from your syllabus. Your actual test will not be as long as this review packet. Unless otherwise indicated, you should simplify all answers by reducing fractions, simplifying radicals, and/or rationalizing denominators (as you've done on your ALEKS homework).

Objective: Determine solutions of two-variable linear equations. (2,3)

1. Find two solutions of $2x - y = 9$. Show that they are indeed solutions.

$(5,1)$ $\quad 2(5)-1 = 10-1 = 9$ ✓

$(7,5)$ $\quad 2(7)-5 = 14-5 = 9$ ✓

2. Find two solutions of $5x + 7y = 70$. Show that they are indeed solutions.

$(0,10)$ $\quad 5(0)+7(10) = 0+70 = 70$ ✓

$(14,0)$ $\quad 5(14)+7(0) = 70+0 = 70$ ✓

3. Find two solutions of $\underbrace{2x = -8}_{x=-4}$. Explain why they are solutions.

TWO SOLUTIONS ARE

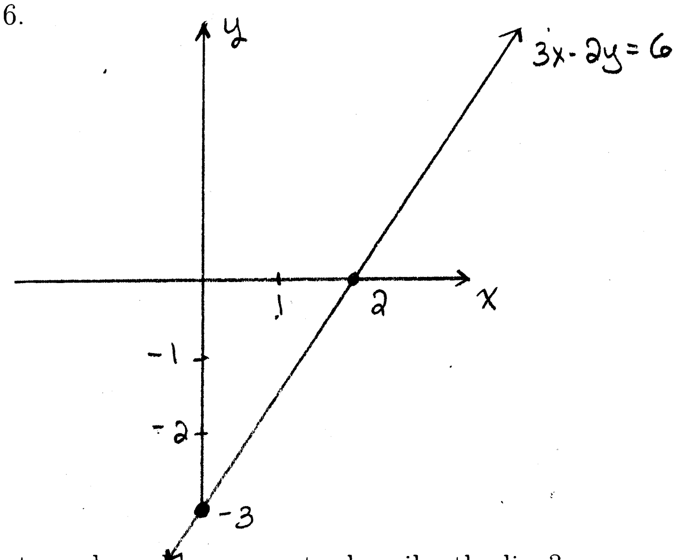
$(-4,0)$ & $(-4,1)$.

EQUATION SAYS X-COORD MUST BE -4; y CAN BE ANY NUMBER.

Objective: Graph a line by finding two points on the line. (2)

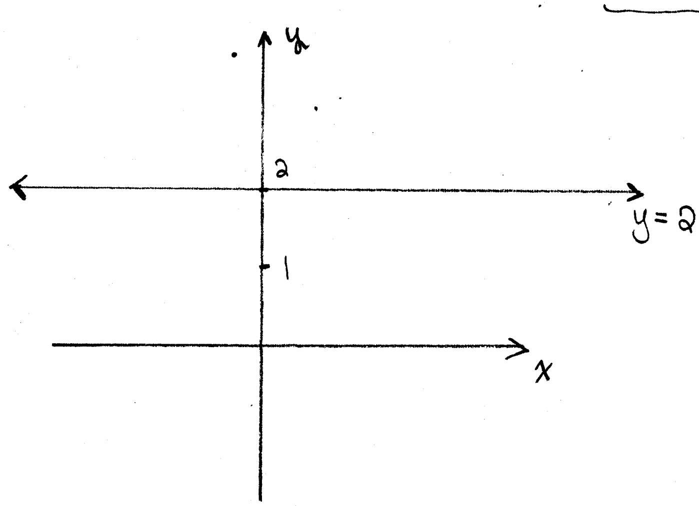
4. Graph the line described by $3x - 2y = 6$.

x	y
2	0
0	-3



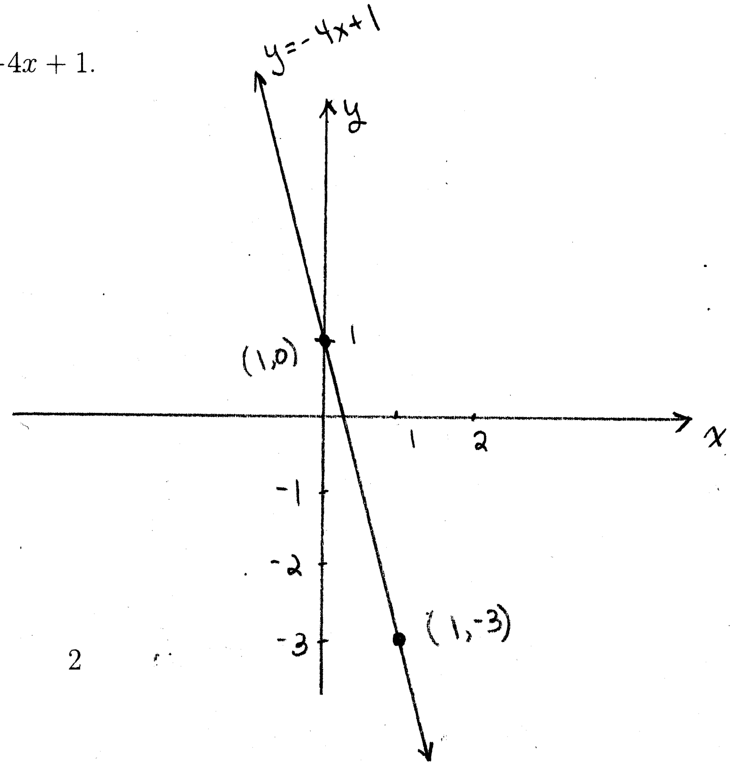
5. Graph the line described by $y = 2$. What word would you use to describe the line?

HORIZONTAL



6. Graph the line described by $y = -4x + 1$.

x	y
0	1
1	-3



Objective: Find the x- and y-intercepts of a line. (2)

7. Determine the x- and y-intercepts of the line described by $4x - 2y = 12$.

X-INT...

$y = 0 \Rightarrow 4x = 12$

$x = 3$

$(3, 0)$ X-INT

Y-INT...

$x = 0 \Rightarrow -2y = 12$

$y = -6$

$(0, -6)$ Y-INT

8. Determine the x- and y-intercepts of the line described by $2x + 3y = 13$.

X-INT...

$y = 0 \Rightarrow 2x = 13$

$x = 13/2$

$(\frac{13}{2}, 0)$ X-INT

Y-INT...

$x = 0 \Rightarrow 3y = 13$

$y = 13/3$

$(0, 13/3)$ Y-INT

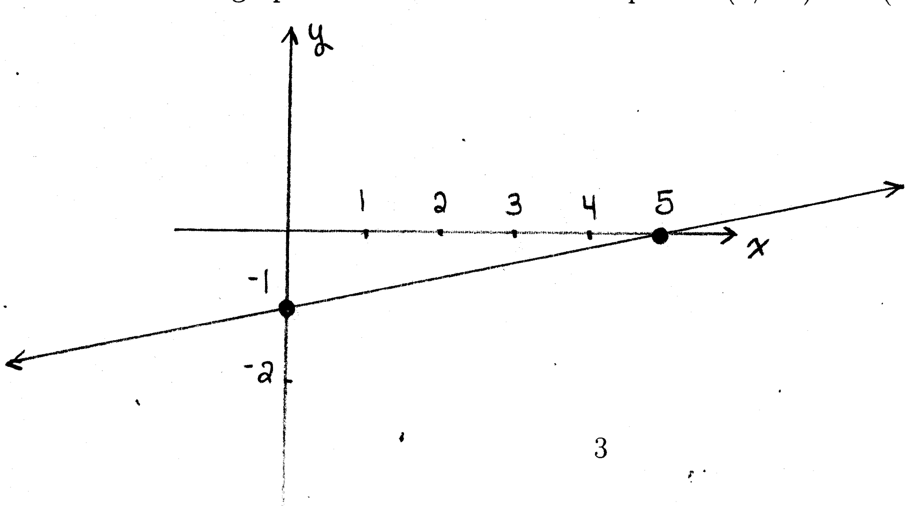
9. Determine the x- and y-intercepts of the line described by $4y = 16$.

$y = 4$ HORIZONTAL LINE!

Y-INT IS $(0, 4)$.

THERE IS NO X-INT.

10. Sketch the graph of the line whose intercepts are $(0, -1)$ and $(5, 0)$.



Objective: Compute the slope of a line and interpret it as a rate of change. (2)

11. Determine the slope of the line that passes through the two points $(4,8)$ and $(-1,-7)$.

$$m = \frac{8-(-7)}{4-(-1)} = \frac{15}{5} = 3$$

12. Determine two points on the line described by the equation $x - 3y = 9$. Then use your points to find the slope of the line.

$X = 0 \Rightarrow -3y = 9$, $y = -3$ $(0,-3)$

$y = 0 \Rightarrow X = 9$ $(9,0)$

$$m = \frac{-3-0}{0-9} = \frac{-3}{-9} = \frac{1}{3}$$

13. The line L passes through the points $(4,6)$ and $(-2,5)$. Find the slope of a line parallel to L. Find the slope of a line perpendicular to L.

Slope of L is

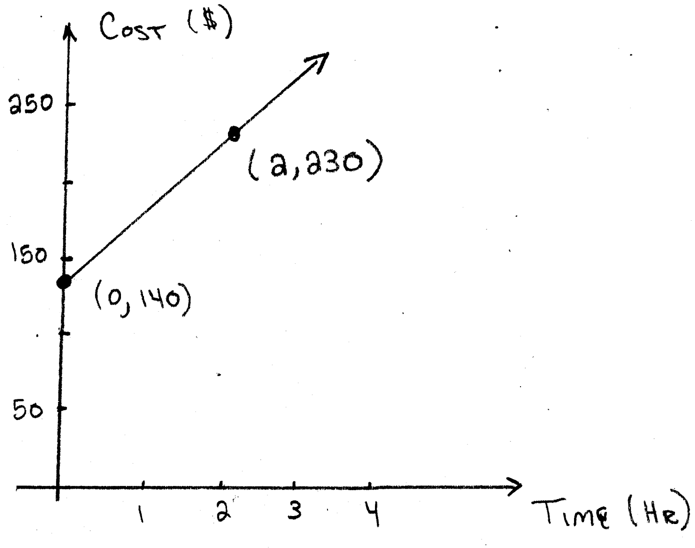
$$\frac{6-5}{4-(-2)} = \frac{1}{6}$$

$$\boxed{m_{\parallel} = \frac{1}{6} \qquad m_{\perp} = -6}$$

14. Sal fixes vintage arcade games. He charges a flat fee of \$140 to make a house call, but then he charges a constant hourly rate on top of that. He recently made a house call to fix a Centipede game and ended up billing the a client \$230 after 2 hours of work. Sketch the graph the shows how much Sal makes versus time (in hours). What does the slope of the graph represent?

$$m = \frac{230-140}{2-0} = \frac{90}{2} = 45$$

m = 45 is the hourly rate.



Objective: Identify equations of horizontal or vertical lines and graph them. (2)

15. Write equations for the horizontal and vertical lines through $(4, -3)$.

HORIZONTAL: $y = -3$

VERTICAL: $x = 4$

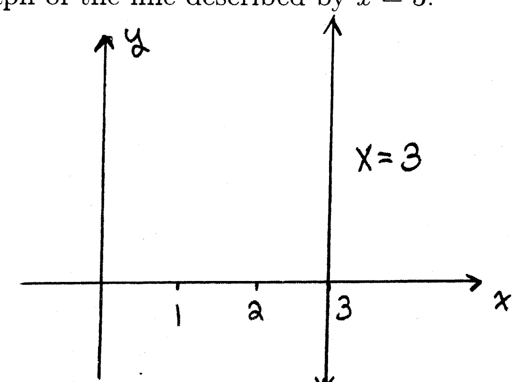
16. The line H passes through the points $(1, 2)$ and $(-1, 2)$. Find an equation of a line parallel to H. Find an equation of a line perpendicular to H.

H IS HORIZONTAL : $y = 2$

ANY OTHER HORIZONTAL LINE IS PARALLEL, e.g., $y = 1$.

ANY VERTICAL LINE IS PERP., e.g., $x = 5$

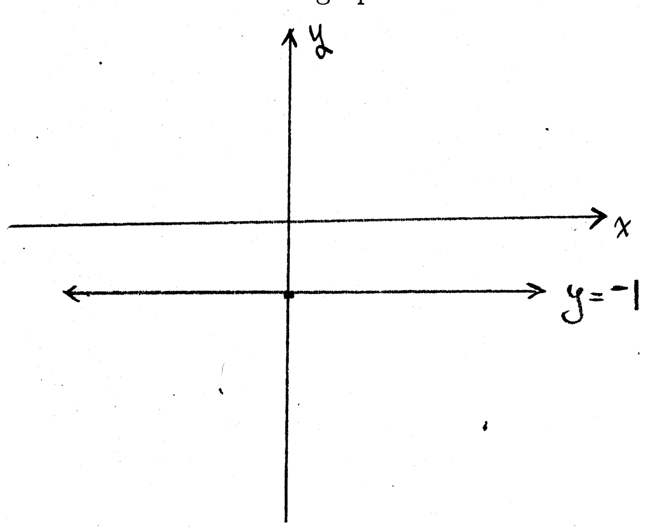
17. Sketch the graph of the line described by $x = 3$.



VERTICAL!

18. Sketch the graph of the line described by $2y = -2$.

$y = -1$ HORIZONTAL!



Objective: Graph parabolas whose equations have the form $y = ax^2$. (8)

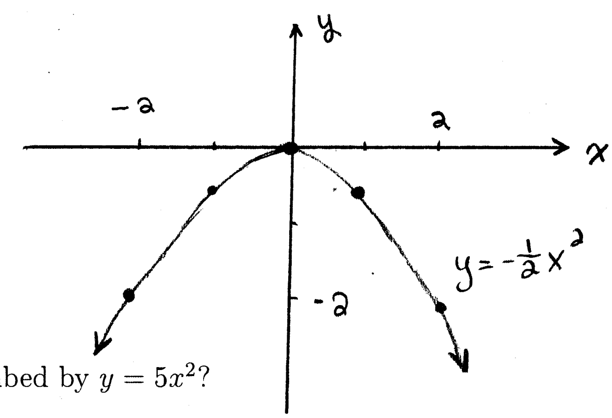
19. Make a table showing five points on the graph of $y = 2x^2$. Include the vertex as one of your five points.

X	$y = 2x^2$
0	0 ← VERTEX
1	2
-1	2
2	8
-2	8

20. Sketch the graph of $y = -\frac{1}{2}x^2$.

X	$y = -\frac{1}{2}x^2$
0	0 ← VERTEX
± 1	$-\frac{1}{2}$
± 2	-2

PARABOLA OPENING DOWNWARD



21. What is the vertex of the parabola described by $y = 5x^2$?

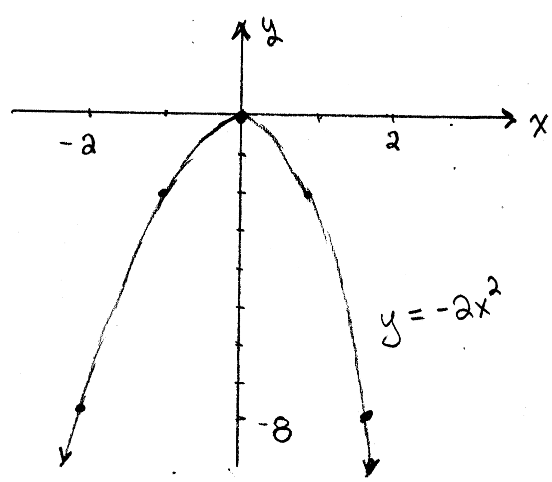
(0, 0)

22. Sketch the graph of $y = -2x^2$.

X	y
0	0
± 1	-2
± 2	-8

VERTEX →

PARABOLA OPENING DOWNWARD



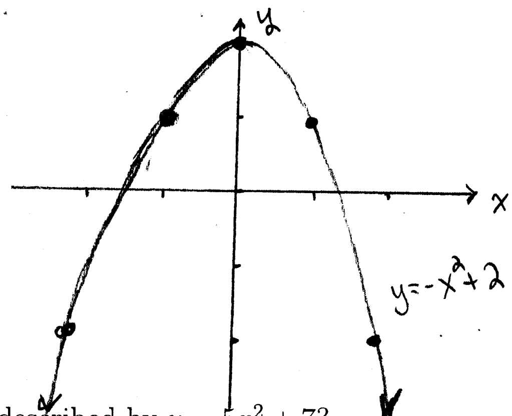
Objective: Graph parabolas whose equations have the form $y = ax^2 + c$. (8)

23. Make a table showing five points on the graph of $y = x^2 - 3$. Include the vertex as one of your five points.

x	$y = x^2 - 3$	
0	-3	← Vertex
1	-2	
-1	-2	
2	1	
-2	1	

24. Sketch the graph of $y = -x^2 + 2$.

	x	y
Vertex →	0	2
	±1	1
	±2	-2

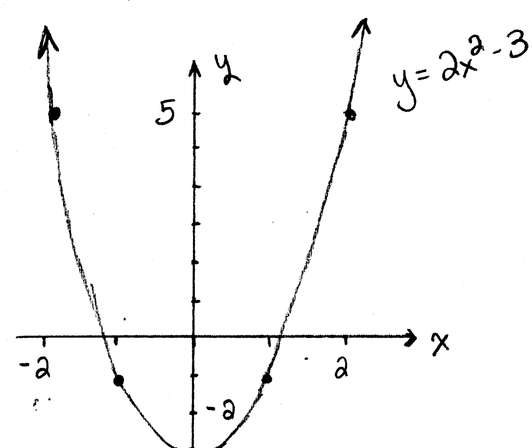


25. What is the vertex of the parabola described by $y = 5x^2 + 7$?

$\boxed{(0,7)}$

26. Sketch the graph of $y = 2x^2 - 3$.

	x	y
Vertex →	0	-3
	±1	-1
	±2	5



Objective: Find and apply the slope-intercept form of the equation of a line. [2]

23. Find the slope and y-intercept of the line described by $2x - 5y = 8$. Write your y-intercept as an ordered pair.

$-5y = -2x + 8$

$y = \frac{2}{5}x - \frac{8}{5}$

SLOPE: $m = \frac{2}{5}$

Y-INT: $(0, -8/5)$

24. A line with slope $-3/7$ has y-intercept $(0, -4)$. Find an equation of the line. Write your final answer in standard form.

$y = -\frac{3}{7}x - 4 \Rightarrow 7y = -3x - 28$

$\Rightarrow \boxed{3x + 7y = -28}$

25. A line is described by the equation $y = 3x - 2$. Find the slope of the line, and determine two points on the line.

SLOPE IS $\boxed{m = 3}$

TWO POINTS... $(1, 1)$, $(0, -2)$

Objective: Apply the point-slope form of the equation of a line. [2]

26. A line with slope 3/5 passes through the point $(10, 8)$. Find an equation for the line.

$y - 8 = \frac{3}{5}(x - 10)$ -OR- $y = \frac{3}{5}x + 2$

27. A line is described by the equation $y + 2 = -4(x - 7)$. Find the slope of the line and a point on the line.

IT'S IN POINT-SLOPE FORM...

SLOPE: $m = -4$

POINT: $(7, -2)$

28. A line passes through the two points $(-4, 3)$ and $(6, -2)$. Find an equation for the line.

$$m = \frac{3-(-2)}{-4-6} = \frac{5}{-10} = -\frac{1}{2}$$

$$y - 3 = -\frac{1}{2}(x+4) \quad \text{or} \quad y = -\frac{1}{2}x + 1$$

Objective: Graph a line using its slope and a point. [2]

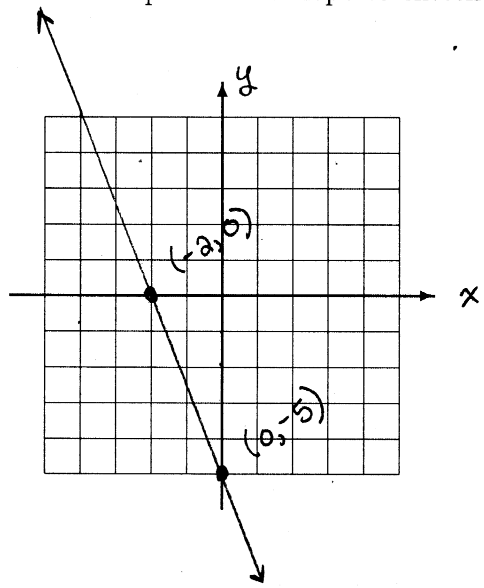
29. A line is described by the equation $5x + 2y = -10$. Rewrite the equation is slope-intercept form. Then use the intercept and the slope to sketch the graph. Be sure to label your axes.

$$2y = -5x - 10$$

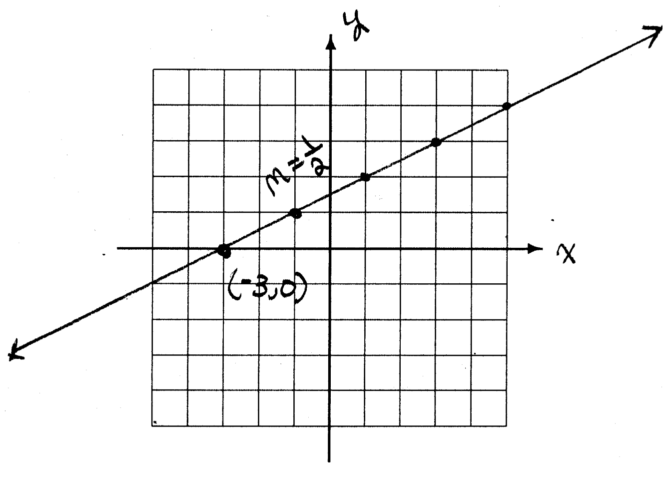
$$y = -\frac{5}{2}x - 5$$

$$m = -\frac{5}{2}$$

$$y\text{-INT} \quad (0, -5)$$



30. A line with slope 1/2 passes through the point $(-3, 0)$. Sketch the graph of the line. Be sure to label your axes.



Objective: Find lines parallel or perpendicular to given lines. [2]

31. A line passes through the points $(1, 1)$ and $(6, 4)$. Find an equation of the perpendicular line through $(5, -6)$.

Slope of given line $= \frac{4-1}{6-1} = \frac{3}{5}$

$m_\perp = -\frac{5}{3}$

$y + 6 = -\frac{5}{3}(x-5)$

or

$y = -\frac{5}{3}x + \frac{7}{3}$

32. A line passes through the point $(-3, -2)$ and is parallel to the line described by $y = 8x - 7$. Find an equation of the line. Write your final answer in standard form.

$m_{\parallel} = 8$

$y + 2 = 8(x+3)$

$y + 2 = 8x + 24$

$-8x + y = 22$

33. A line passes through the points $(3, 1)$ and $(3, 0)$. Find equations of the lines parallel and perpendicular to the original line. Label which is which.

Vertical line.

Parallel : $x = 5$ ← Vertical

Perp : $y = 5$ ← Horizontal

Objective: Apply lines and linear equations in real-world applications. [2]

34. The length of the humerus (the bone from the elbow to the shoulder) is a good indicator of height. A female with a humerus of length 26.1 cm is approximately 143.5 cm tall, while a female with a 20.4 cm humerus is about 127.6 cm tall. Assume that humerus length and height satisfy a linear equation. Determine that equation. Round all numbers to the nearest tenth.

$(26.1, 143.5)$

$(20.4, 127.6)$

$m = \frac{143.5 - 127.6}{26.1 - 20.4} = \frac{15.9}{5.7}$

≈ 2.8

Using $m = 2.8$ and $(26.1, 143.5)$

$y - 143.5 = 2.8(x - 26.1)$

or

$y = 2.8x + 70.4$

35. A car currently worth \$24,575 depreciates at a constant rate of \$1752. Let v represent the value of the car in dollars, and let t represent time in years. Using the variables v and t, write an equation for the value of the car.

$V = -1752t + 24575$

Objective: Determine whether a relation is a function. [1]

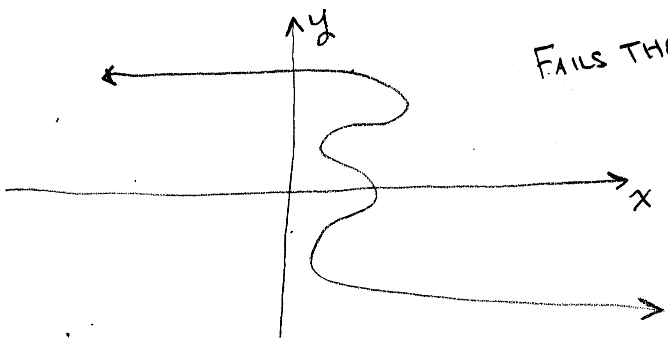
36. Carefully explain why this relation is not a function.

$$\{(1,2),(2,5),(3,8),(4,10),(-1,8),(3,9)\}$$

X = 3

HAS TWO DIFFERENT y-VALUES.

37. Sketch the graph of a relation that is not a function.



FAILS THE VERTICAL LINE TEST.

38. For any real number, x, let $f(x) = x^3 - x^2 + 1$. Does this define a function? How do you know?

YES. EACH SINGLE X-INPUT PRODUCES EXACTLY ONE y-OUTPUT, f(x).

39. Does this table describe a function? How do you know?

x	-2	2	-5	8	7	13
y	1	1	1	1	1	1

YES, NO X-COORD IS REUSED.
SO, EACH X GIVES EXACTLY ONE y.

Objective: Determine the domain and range of a function. [1]

40. What is the domain of the function $F(x) = x^2 + |x|$?

ALL REAL NUMBERS

$(-\infty, \infty)$

41. What is the domain of the function $g(x) = \dfrac{x^2+x-6}{\underbrace{x^2+6x+5}}$?

$(x+1)(x+5) = 0$

$x = -1 \quad x = -5$

$\boxed{\text{DOMAIN} = \text{ALL REAL \#'s EXCEPT } x = -1 \text{ \& } x = -5}$

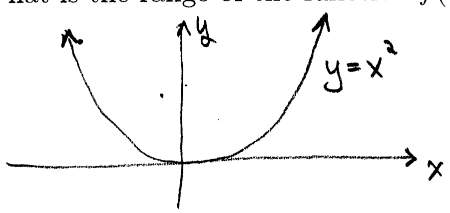
42. What is the domain and range of the relation defined by the following table of values?

x	-2	2	-5	8	7	13
y	1	1	1	1	1	1

DOMAIN = $\{-5, -2, 2, 7, 8, 13\}$

RANGE = $\{1\}$

43. What is the range of the function $f(x) = x^2$?



$\boxed{\text{RANGE} = [0, \infty)}$

Objective: Use function notation and evaluate functions. [1,5]

44. Let $f(x) = \sqrt[4]{x+7}$. Evaluate $f(9)$. What about $f(-8)$?

$f(9) = \sqrt[4]{16} = \boxed{2}$

$f(-8) = \sqrt[4]{-1}$ $\boxed{\text{NOT DEFINED!}}$

45. Let $g(y) = 2y^2 - 3y + 7$. Evaluate $g(-5)$.

$g(-5) = 2(-5)^2 - 3(-5) + 7 = 2(25) + 15 + 7$

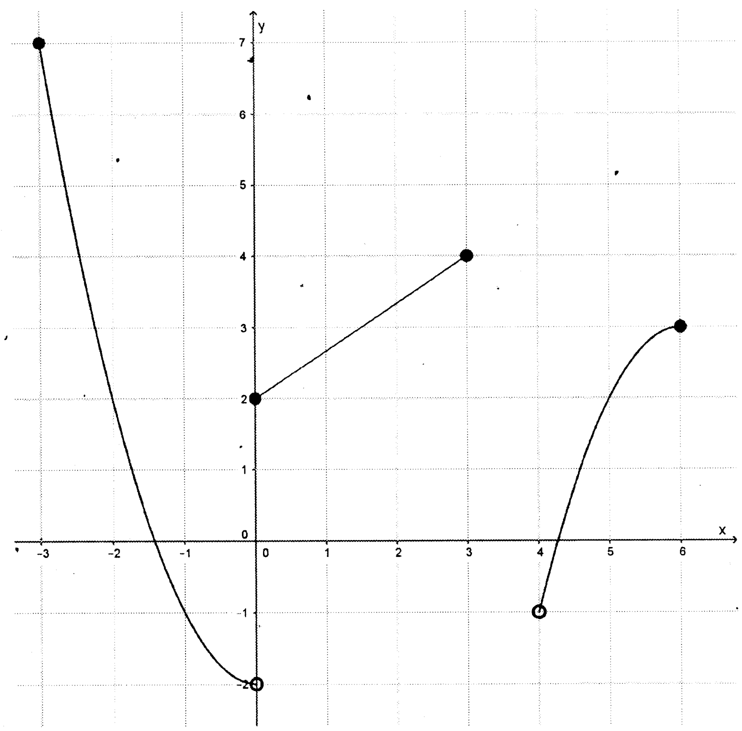
$= \boxed{72}$

46. Let $G(x) = \dfrac{2x}{2x-8}$. Evaluate $G(4)$.

$\boxed{G(4) \text{ IS NOT DEFINED}}$ BECAUSE IT WOULD CAUSE DIVISION BY ZERO.

Objectives: Interpret graphs of functions. Given the graph of a function, determine where the function is positive, negative, or zero; determine intervals on which the function is increasing, decreasing, or constant; and determine the local maxima and minima. [5]

47. The graph of $y = h(x)$ is shown below. Use the graph to solve each part of this problem.



(a) Is this the graph of a function? How do you know?

YES, THE GRAPH PASSES THE VERTICAL LINE TEST.

(b) What is the domain of h?

$[-3,3] \cup (4,6]$

(c) What is the range of h?

$(-2, 7]$

(d) Determine $h(-2)$.

$h(-2) = 2$

(e) Determine $h(3.5)$.

$h(3.5)$ IS NOT DEFINED.

(f) Determine $h(0)$.

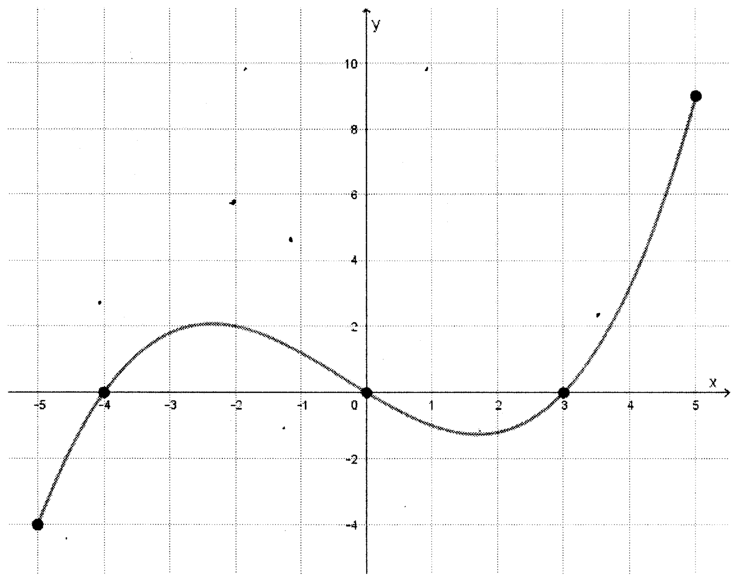
$h(0) = 2$

(g) Determine an x-value for which $h(x) = 3$. How many are there?

3 OF THEM ...

$X \approx -2.25, \quad X \approx 1.5, \quad X = 6$

48. The graph of $y = f(x)$ is shown below.



(a) What is the domain of f?

$[-5,5]$

(b) What is the range of f?

$[-4,9]$

(c) Determine intervals on which $f(x) < 0$.

$[-5,-4) \cup (0,3)$

(d) Determine intervals on which $f(x) > 0$.

$(-4,0) \cup (3,5]$

(e) Determine open intervals on which f is increasing.

$(-5,-2.5) \cup (1.5,5)$

(f) Determine open intervals on which f is decreasing.

$(-2.5,1.5)$

(g) Determine any relative (local) minimum values and maximum values.

REL. MAX:

$y = 2$

WHERE $X \approx -2.5$

REL. MIN:

$y \approx -1$.

WHERE $X \approx 1.5$

Objective: Simplify difference quotients.

49. Let $f(x) = 5 - 2x$. Expand and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{[5-2(x+h)]-[5-2x]}{h} = \frac{5-2x-2h-5+2x}{h}$$

$$= -\frac{2h}{h} = \boxed{-2}$$

50. Let $g(x) = x^2 - 3x + 7$. Expand and simplify the difference quotient $\dfrac{g(x+h)-g(x)}{h}$.

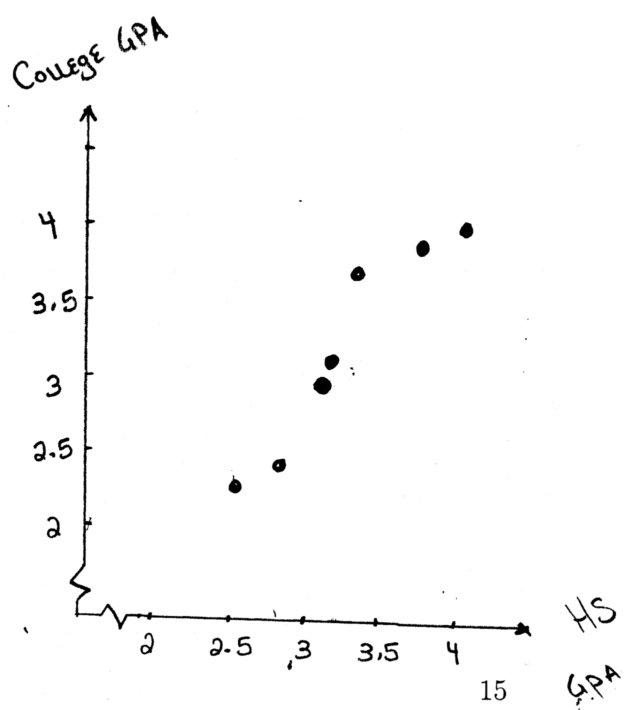
$$\frac{g(x+h)-g(x)}{h} = \frac{[(x+h)^2-3(x+h)+7]-[x^2-3x+7]}{h} = \frac{x^2+2xh+h^2-3x-3h+7-x^2+3x-7}{h}$$

$$= \frac{2xh+h^2-3h}{h} = \frac{h(2x+h-3)}{h} = \boxed{2x+h-3}$$

Objective: Construct a scatterplot. [4]

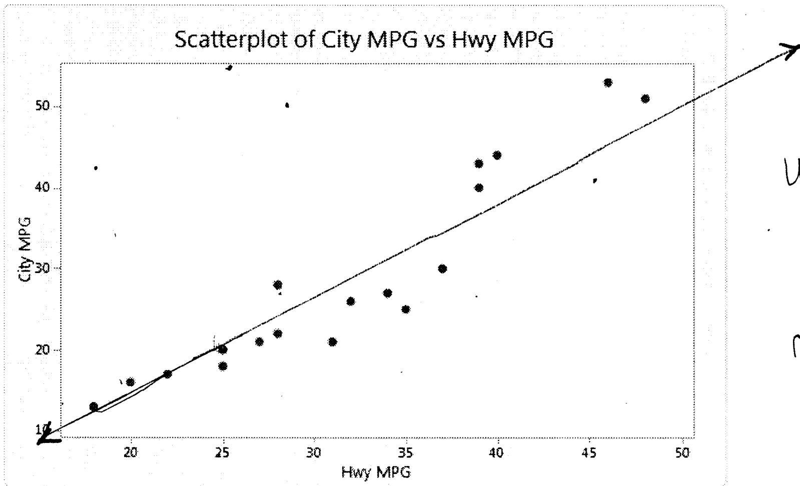
51. In the following (x, y) ordered pairs, x represents a student's high school GPA, and y represents the same student's college GPA. Sketch the scatterplot.

(3.2, 3.1), (2.5, 2.2), (3.6, 3.8), (3.2, 3.6), (3.1, 3.0), (2.8, 2.4), (4.0, 3.9)



Objective: Recognize a linear relationship, and determine an equation that describes the relationship. [2,4]

52. A scatterplot is shown below. Sketch the best fit line. Then use two points on your line to determine an equation of the line. Write your answer in slope-intercept form.



Using (25, 20) & (40, 35) ...

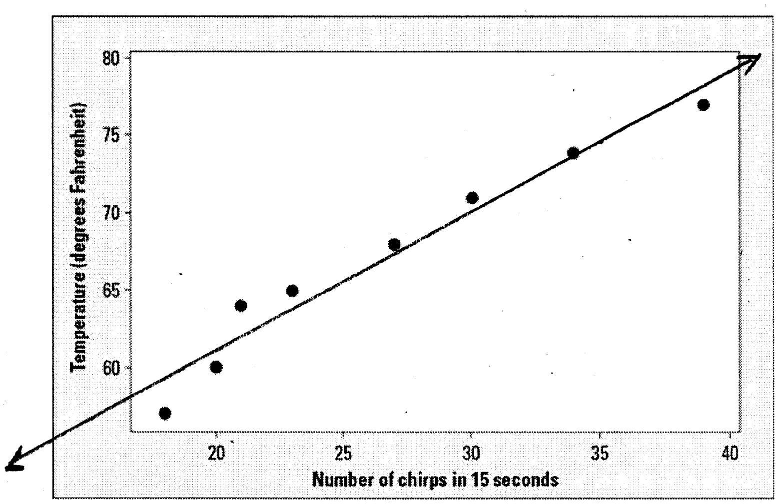
$m = \frac{15}{15} = 1$

$y - 20 = 1(x - 25)$

or

$\boxed{y = x - 5}$

53. A scatterplot is shown below. Sketch the best fit line. Then use two points on your line to determine an equation of the line. Write your answer in slope-intercept form.



Using (20, 61) AND (35, 74) ...

$m = \frac{13}{15}$ $\quad y - 61 = \frac{13}{15}(x - 20)$

ABOUT $y = 0.87x + 43.67$

Objective: Use an equation to make predictions in a linear relationship. [2,4]

54. Look at the equation you determined in problem #52 above. Use your equation to predict the City MPG of a car whose Hwy MPG is 44.

$x =$ Hwy MPG

$y =$ City MPG

$y = x - 5$

$\boxed{y = 44 - 5 = 39}$

55. Look at the equation you determined in problem #52 above. Use your equation to predict the Hwy MPG of a car whose City MPG is 35.

$35 = x - 5$

$\Rightarrow \boxed{x = 40}$

56. Look at the equation you determined in problem #53 above. Use your equation to predict the temperature if there are 25 chirps in 15 ~~minutes~~ SECONDS.

$x =$ # OF CHIRPS

$y =$ TEMP °F

$y = 0.87x + 43.67$

$x = 25 \Rightarrow y = 0.87(25) + 43.67$

$\approx \boxed{65.4°F}$

57. Look at the equation you determined in problem #53 above. Use your equation to predict the number of chirps in 15 ~~minutes~~ SECONDS if the temperature is 70°F.

$70 = 0.87x + 43.67$

$\Rightarrow x \approx 30.3$

$\boxed{\text{ABOUT 30 CHIRPS}}$

Objective: Sketch the graph of $f(x) = ax + b$. [2,3]

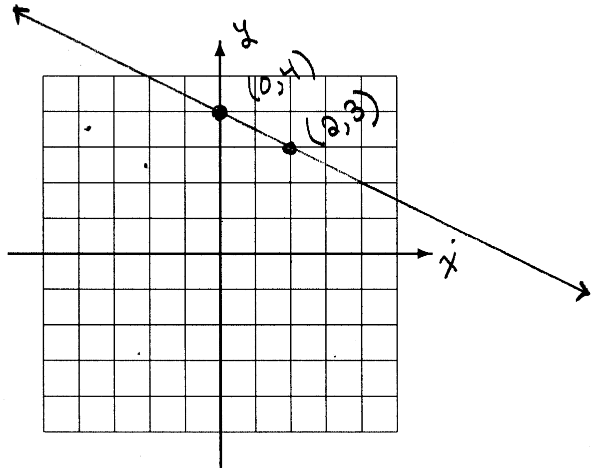
58. Determine two points on the graph of $f(x) = -\frac{1}{2}x + 4$. Then sketch the graph. Be sure to label your axes.

$X = 0$

$\Rightarrow y = f(0) = 4$

$X = 2$

$\Rightarrow y = f(2) = 3$



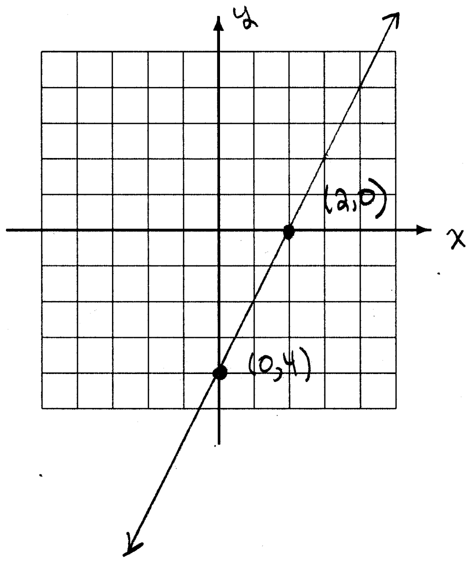
59. Sketch the graph of $g(x) = 2x - 4$. Be sure to label your axes.

$X = 0$

$\Rightarrow y = g(0) = -4$

$X = 2$

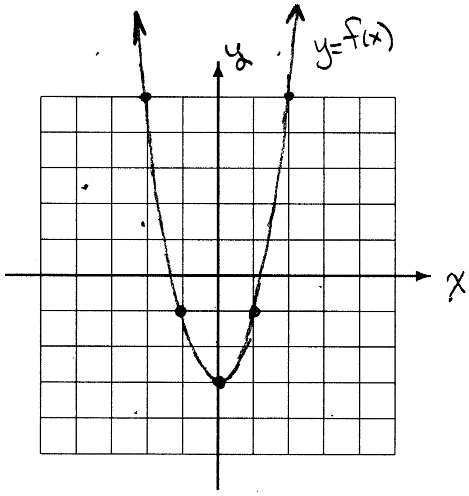
$\Rightarrow y = g(2) = 0$



Objective: Sketch the graph of $f(x) = ax^2 + c$. [8]

60. Determine five points on the graph of $f(x) = 2x^2 - 3$. Then plot your points and sketch the graph.

x	y = f(x)
0	-3
±1	-1
±2	5



61. Determine five points on the graph of $g(x) = 4 - \frac{2}{3}x^2$. Then plot your points and sketch the graph.

x	y = g(x)
0	4
±1	$\frac{10}{3} = 3\frac{1}{3}$
±2	$\frac{4}{3} = 1\frac{1}{3}$