Math 109 - Test 2A

March 12, 2020

Name **key**

Score _____

Show all work to receive full credit. Supply explanations where necessary. Label your axes when graphing.

1. (4 points [2]) Find equations for the horizontal and vertical lines passing through $(5, -11)$. Label which is which.

VERTICAL

$X = 5$

HORIZ.

$y = -11$

2. (8 points [2,3]) Think about the linear equation $-3x + 4y = 8$ and its graph.

(a) Determine the x- and y-intercepts.

X-INT... $y = 0 \Rightarrow -3x = 8$

$x = -\frac{8}{3}$

$\left(-\frac{8}{3}, 0\right)$

Y-INT... $x = 0 \Rightarrow 4y = 8$

$y = 2$

$(0, 2)$

(b) Find the slope.

$$m = \frac{2-0}{0 + \frac{8}{3}} = \frac{2}{\frac{8}{3}} = \frac{6}{8} = \frac{3}{4}$$

(c) Carefully sketch the graph and label your axes.
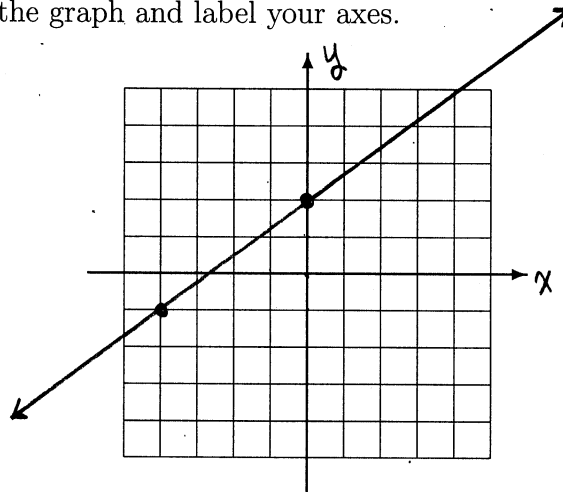
3. (8 points [8]) Think about the graph of the equation $y = -\frac{1}{2}x^2 + 3$.
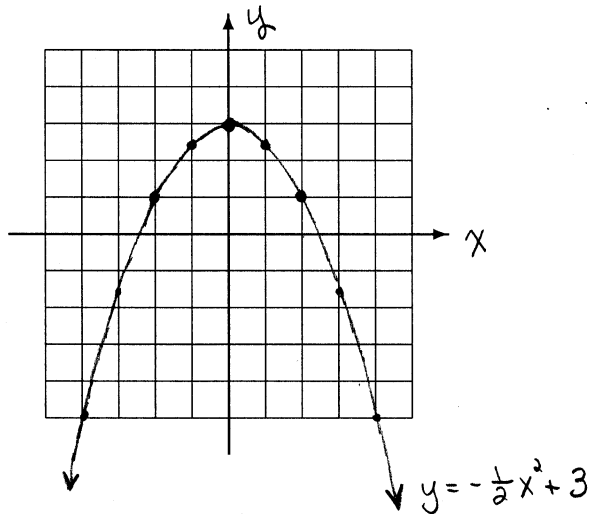
   (a) What is the name of this type of graph?

   PARABOLA

   (b) Make a table showing five (5) points on the graph. Include the vertex as one of your points (and say which one is the vertex).

X	y
0	3	← VERTEX
1	2.5
-1	2.5
2	1
-2	1

   (c) Carefully sketch the graph and label your axes.

   $y = -\frac{1}{2}x^2 + 3$

4. (6 points [2]) A line passes through the two points $(2, -3)$ and $(4, 7)$. Find an equation for the line. Write your final answer in slope-intercept form.

   $m = \dfrac{7-(-3)}{4-2} = \dfrac{10}{2} = 5$

   POINT : $(4, 7)$

   $y = 5x - 13$

   $y - 7 = 5(x - 4)$

 $y - 7 = 5x - 20$

2

5. (8 points) Let $f(x) = x^2 + x$. Expand and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h) - f(x)}{h} = \frac{\left[(x+h)^2 + (x+h)\right] - \left[x^2 + x\right]}{h} = \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$$

$$= \frac{2xh + h^2 + h}{h} = \frac{h(2x + h + 1)}{h} = \boxed{2x + h + 1}$$

6. (7 points [2,5]) Bartlep's Heating and Cooling charges a flat fee of \$110 to make a house call, but then charges a constant hourly rate on top of that. A technician recently made a house call to fix a furnace and ended up billing the client \$190 after 2 hours of work.

(a) Sketch the graph the shows client cost (in dollars) versus time (in hours). Label your axes. (Your vertical axis should be the cost axis.)
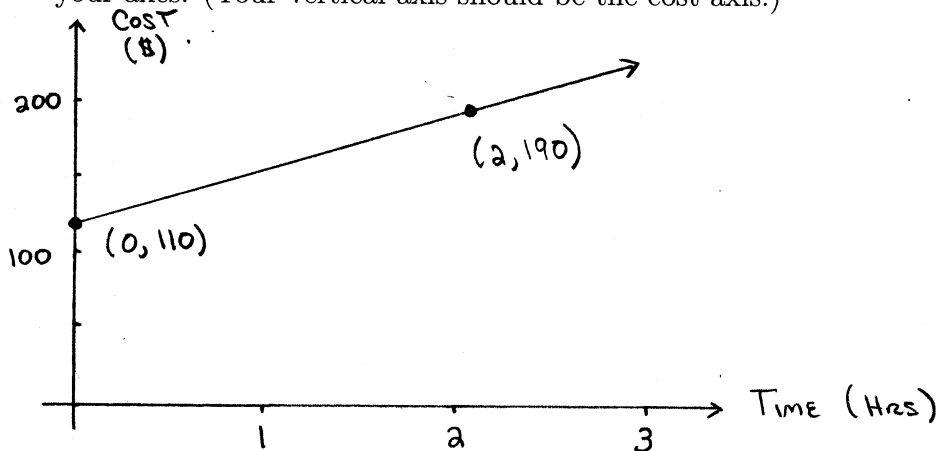
(b) Which single word or phrase in the problem situation indicates that the graph should be a line?

CONSTANT HOURLY RATE

(c) Compute the slope of the graph. What does the slope of the graph represent?

$$m = \frac{190 - 110}{2 - 0} = \frac{80}{2} = \boxed{40}$$

HOURLY RATE = \$40 per HOUR

3

7. (6 points [2]) The line L passes through the point $(2, -5)$ and is perpendicular to the line described by $2x + 4y = 11$. Determine an equation for L. Write your final answer in standard form.

$$4y = -2x + 11$$
$$y = -\frac{2}{4}x + \frac{11}{4}$$
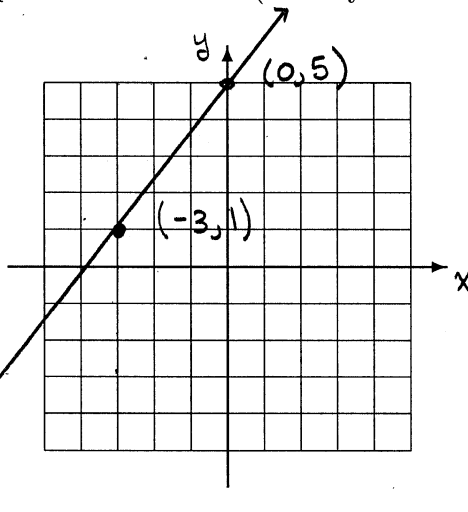$$m = -\frac{1}{2}$$
$$m_\perp = 2$$

$$y + 5 = 2(x - 2)$$
$$y + 5 = 2x - 4$$
$$\boxed{-2x + y = -9}$$

8. (6 points [2]) A line with slope 4/3 passes through the point $(-3, 1)$. Sketch the graph of the line, and find an equation of the line. (Label your axes.)

$$\text{Slope } \frac{4}{3} = \frac{RISE}{RUN}$$

$$Y\text{-INT } (0, 5)$$

$$\boxed{y = \frac{4}{3}x + 5}$$

$(0, 5)$

$(-3, 1)$

9. (3 points [1]) Three relations are shown below. Circle all that are NOT functions. Then write a sentence explaining why you made your choice(s).

(a) $\{(1, 0), (1, 0), (0, 1), (0, 1)\}$

(b) $\{(3, 4), (1, -4), (-3, 4), (-1, 4), (2, 4), (2, -4)\}$

(c) $\{(x, y) : x \text{ is a whole number and } y = 2x\}$

EACH X-COORD HAS A SINGLE y-COORD EXCEPT FOR

10. (3 points [1]) Determine the domain of $f(x) = \dfrac{x - 7}{3x - 10}$.

$$\boxed{x \neq \frac{10}{3}}$$

$$3x - 10 = 0 \implies x = \frac{10}{3}$$

DENOM CANNOT BE ZERO.

DOMAIN = SET OF ALL X FOR WHICH $X \neq \frac{10}{3}$

4

11. (3 points [1]) Determine the range of $f(x) = x^2 - 5$. Briefly explain your reasoning.

THE graph OF f IS A PARABOLA, OPENING UPWARD, WITH VERTEX AT $(0, -5)$.

RANGE $= [-5, \infty)$

12. (3 points [1]) What is the domain of the function $g(x) = 5x^2 + |x|$?

ALL REAL NUMBERS

13. (3 points [2,4]) Biologists studying crickets found a strong positive linear relationship between temperature and the number of chirps per 15 seconds. Their best fit line satisfies

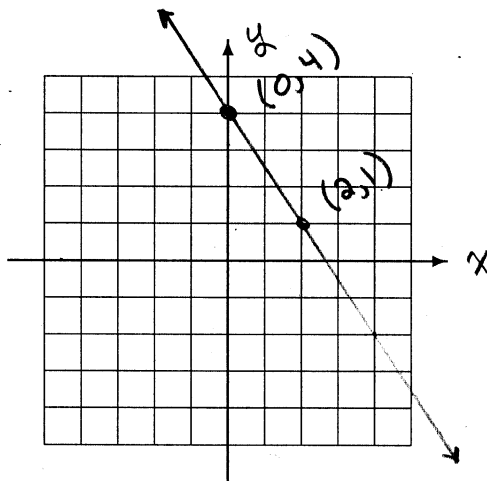Temperature in °F $= 0.87 \times$ (Number of chirps in 15 seconds) $+ 43.7$.

About how many chirps should you expect in 15 seconds if the temperature is 70°F?

$70 = 0.87x + 43.7$

$26.3 = 0.87x \implies x = 30.23$

ABOUT 30 CHIRPS

14. (6 points [2,3]) Sketch the graph of the function $f(x) = -\dfrac{3}{2}x + 4$. Label your axes and two points on the graph.

y-INT IS $(0,4)$

SLOPE IS $-\dfrac{3}{2}$.

STARTING AT $(0,4)$

go 3 DOWN AND 2 RIGHT TO get TO $(2,1)$

5

15. (12 points [2,4]) A manager believes that employees produce more if they have high dexterity skills. She compares employees dexterity test scores to their hourly productivity. For each ordered pair below, the 1st coordinate represents test score and the 2nd coordinate represents number of units produced per hour.

$$(12, 55), (14, 63), (17, 67), (16, 70), (11, 51)$$

(a) Sketch the scatterplot corresponding to the data. Label your axes.
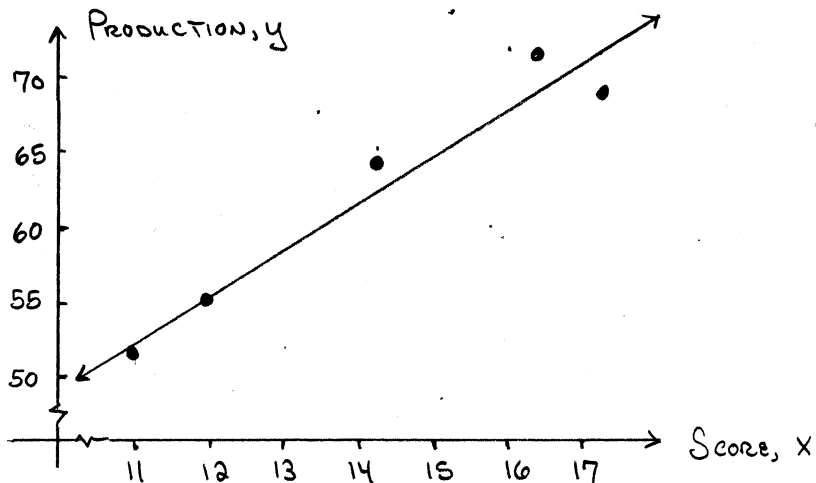
(b) Sketch a line that approximates the best fit. Then find an equation for your line. Round all numbers to the nearest tenth.

I'LL USE POINTS
$(12, 55)$ ¢ $(17, 68)$

$$m = \frac{68-55}{17-12} = \frac{13}{5} = 2.6$$

$$y - 55 = 2.6(x - 12)$$

or

$$y = 2.6x + 23.8$$

(c) Use your equation to approximate the number of hourly units produced by an employee who scores 20.
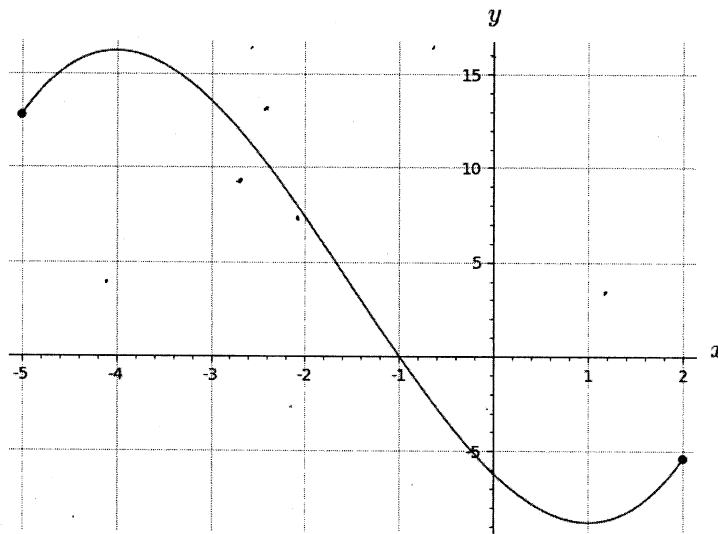
$$y = 2.6(20) + 23.8$$
$$= 75.8$$

ABOUT 76

(d) Use your equation to approximate the test score of an employee who produces 45 units per hour.

$$45 = 2.6x + 23.8$$

$$21.2 = 2.6x \implies x = 8.15$$

ABOUT 8

6

16. (14 points [1,5]) The graph of $y = f(x)$ is shown below. Use the graph to solve each part of this problem.

(a) Is this the graph of a function? How do you know?

YES, THE graph PASSES THE VERTICAL LINE TEST.

(b) What is the domain of f?

$[-5, 2]$

(c) What is the range of f?

ABOUT $[-9, 16]$

(d) Determine $f(0)$.

$f(0) = -6$

(e) Determine the interval(s) on which $f(x) < 0$.

$(-1, 2]$

(f) Determine open intervals on which f is decreasing.

ABOUT $(-4, 1)$

(g) Determine the relative minimum value(s) of f.

$y \approx -9$ AT $X = 1$