Math 112 - Test 3
November 17, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) $850 is deposited into an account earning 3.5% simple interest. The account is closed after 8.75 years.

   (a) How much interest does the account earn?

$$I = Prt = (850)(0.035)(8.75)$$
$$= \$260.31$$

   (b) What is the total value of the account when it is closed?

$$A = 850 + 260.31$$
$$= \$1110.31$$

2. (5 points) Sam deposited $1200 into an account earning simple interest. After 6 years, he closed the account and had $1650. What was the simple interest rate? Write your result as a percent.

$$I = 1650 - 1200 = 450$$

$$450 = 1200(r)(6) = 7200$$

$$r = \frac{450}{7200} = 0.0625 \Rightarrow 6.25\%$$

3. (5 points) You have a choice of two simple interest loans: 5% for 6 years or 6% for 5 years. Which is better and why?

5% for 6 years

$I = P(0.05)(6)$
$= 0.3P$

6% for 5 years

$I = P(0.06)(5)$
$= 0.3P$

EXACT SAME AMOUNT OF INTEREST!

4. (8 points) A couple decides to set aside \$6,000 in a savings account for a trip. Interest is compounded quarterly at 8%.

   (a) How much money is in the account after 10 years?

$$A = 6000\left(1 + \frac{0.08}{4}\right)^{(4)(10)} \approx \$13248.24$$

   (b) How much money was made in interest?

$$A - P = \$7248.24$$

5. (6 points) Determine the better investment: 7% compounded monthly or 7.2% compounded semiannually. (Compute and compare the effective interest rates.)

7.2% MONTHLY: $\left(1 + \frac{0.07}{12}\right)^{12} - 1 \approx 0.0723$

7.2% SEMI: $\left(1 + \frac{0.072}{2}\right)^{2} - 1 \approx 0.0733$ ★

THE 7.2% SEMIANNUALLY RATE IS A BETTER INVESTMENT

6. (6 points) Compute each of the following.

   (a) $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 720$

   (b) $\frac{100!}{98!\,2!} = \frac{100 \cdot 99}{2} = 50 \cdot 99 = 4950$

   (c) ${}_7P_5 = \frac{7!}{2!} = 2520$

7. (8 points) Suppose you open an annuity with quarterly payments of \$400 at 5% compounded quarterly for 15 years.

   (a) Find the future value of the annuity.

$$A = \frac{400\left[\left(1+\frac{0.05}{4}\right)^{(4)(15)} - 1\right]}{\left(\frac{0.05}{4}\right)} \approx \$35,429.80$$

   (b) How much interest will you earn?

$$35429.80 - 60(400) = \$11,429.80$$

8. (6 points) Suppose you begin depositing monthly payments into an account earning 10% compounded monthly. Your goal is to accumulate \$15,000 in 5 years. What should your monthly payments be?

$$R = \frac{15000\left(\frac{0.10}{12}\right)}{\left[\left(1+\frac{0.10}{12}\right)^{(12)(5)} - 1\right]} \approx \$193.71$$

9. (6 points) Sarah has learned that she can get a new computer by agreeing to make monthly payments of \$68 for five years. After reading the fine print, she realized that these monthly payments include a finance charge of 16.99% compounded monthly. How much would the computer cost Sarah if she paid all at once in cash?

$$P = \frac{68\left[1-\left(1+\frac{0.1699}{12}\right)^{(-12)(5)}\right]}{\left(\frac{0.1699}{12}\right)} \approx \$2736.73$$

10. (18 points) A house sells for \$212,400 and a 15% down payment is made. A mortgage is secured for 30 years at 3.85% compounded monthly.

(a) What amount is financed?

15% OF \$212,400 IS \$31,860

$\Rightarrow$ AMOUNT FINANCED = \$180,540.00

(b) What is the monthly payment?

$$R = \frac{180540\left(\frac{0.0385}{12}\right)}{\left[1-\left(1+\frac{0.0385}{12}\right)^{-360}\right]} \approx \$846.39$$

(c) When the loan is paid off in 30 years, what will be the total interest paid?

$360(846.39) - 180540$

$= \$124,160.40$

(d) Compute the first 3 rows of the amortization schedule. Include the interest, amount paid to principal, and the outstanding balance.

STARTING BALANCE = 180540, PAYMENTS = 846.39

PAYMENT #	INTEREST PAID	PRINCIPAL PAID	BALANCE
1	579.23	267.16	180,272.84
2	578.38	268.01	180,004.83
3	577.52	268.87	179,735.96

11. (3 points) A student buying school supplies can choose from 8 different notebooks and 12 different binders. How many notebook/binder combinations are there?

$8 \times 12 = 96$

12. (5 points) Four lower case letters of the English alphabet are selected **without replacement** to form a personal identification code. How many different 4-letter codes are possible?

$\underbrace{26 \times 25 \times 24 \times 23}_{{}_{26}P_4} = 358800$

13. (5 points) A scientist is labeling his samples with 7-letter ordered sequences consisting of 4-X's, 2-Y's and 1-Z (XYYXZXX, for example). How many different sequences are possible?

$$\frac{7!}{4!\,2!\,1!} = \frac{7 \times 6 \times 5}{2} = 105$$

14. (6 points) There are 50 U. S. Senators. A four person committee is to be formed.

(a) How many 4-person committees are possible?

${}_{50}C_4 = 230,300$

(b) How many of those 4-person committees include Senator Dick Durbin?

ONLY 1 WAY TO CHOOSE DICK DURBIN

${}_{49}C_3 = 18,424$ WAYS TO CHOOSE OTHER 3

$1 \cdot {}_{49}C_3 = 18,424$

15. (8 points) **Take-Home Problem**
At age 30, Fred is opening a retirement account that will begin to pay distributions when Fred is 64. The account will earn 7% compounded monthly, and Fred would like to have monthly distributions of $2100 for 20 years. How much must Fred deposit per month (from now until he is 64) in order to meet his goals?

At age 64, Fred needs

$$P = \frac{2100\left[1-\left(1+\frac{0.07}{12}\right)^{-(12)(20)}\right]}{\left(\frac{0.07}{12}\right)}$$

For 20 years of $2100 per month

$$\approx \boxed{\$270{,}863.26}$$

Payments to reach this...

$$R = \frac{270863.26\left(\frac{0.07}{12}\right)}{\left[\left(1+\frac{0.07}{12}\right)^{(12)(34)}-1\right]}$$

Monthly payments for 34 years

$$\approx \boxed{\$162.38}$$

↑ Monthly payments