Math 112 - Final Exam

December 15, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (2 points) Let X be the set of letters of the words *TACO CAT*. Write X in roster notation.

$X = \{T, A, C, O\}$

2. (2 points) Let A be the set of all good Hollywood actors. Is this set well defined? Explain why or why not.

No, there is no agreed upon definition of "good", so we cannot determine if a particular actor is in A.

3. (6 points) The set B is defined below using set-builder notation.

$$B = \{x | \ x \in \mathbb{N} \text{ and } x < 4\}$$

(a) Write B is roster notation.

$\{1, 2, 3\}$

(b) What is the cardinality of B?

$n(B) = 3$

(c) Give an example of a set that is equivalent to B, but not equal to B.

$\{a, b, c\}$

4. (4 points) List all subsets of the set $\{x, y\}$.

$\emptyset$, $\{x\}$, $\{y\}$, $\{x, y\}$

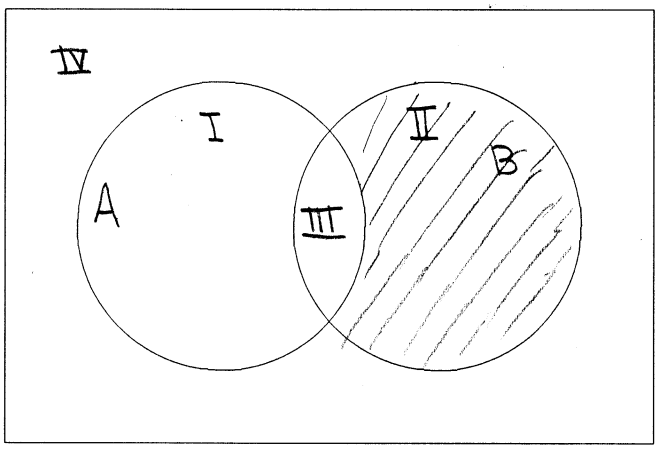
5. (10 points) Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$.

   (a) Determine the set $A - B$. $\{1, 3\}$

   (b) Determine $n(A \times B)$. $4 \times 4 = 16$

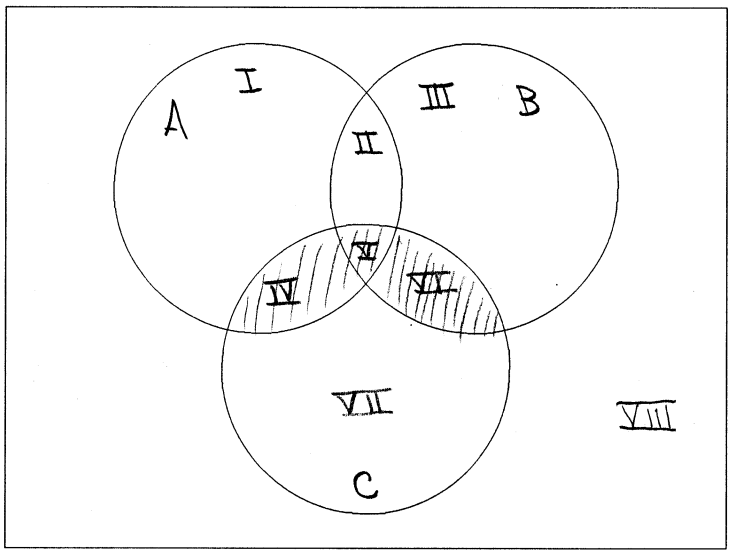
   (c) Determine the set $B \cap \emptyset$. $\emptyset$

6. (4 points) Label the circles and shade the region corresponding to $A' \cap B$.



$\{II, IV\} \cap \{III, II\}$
$= \{II\}$

7. (6 points) Label the circles and shade the region corresponding to $(A \cup B) \cap C$.

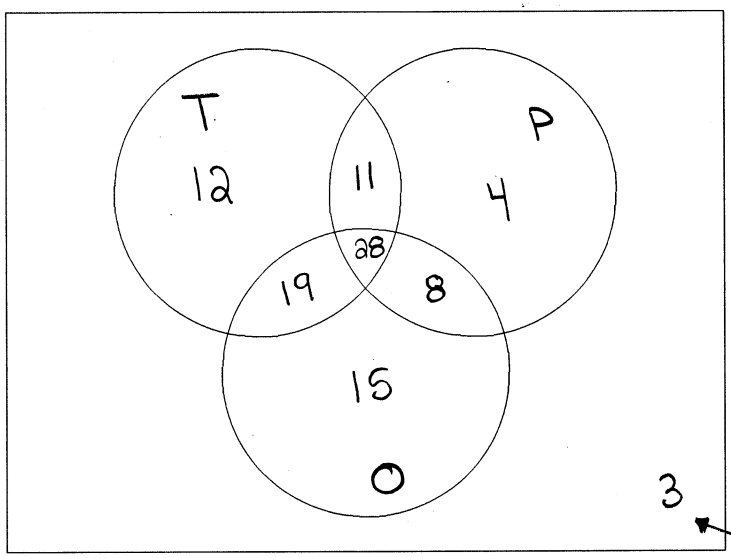


$\{I, II, III, IV, V, VI\}$
$\cap \{IV, V, VI, VII\}$
$= \{IV, V, VI\}$

8. (12 points) One hundred professionals were asked how they obtained their daily news. Here are the results:

- 70 got their news from television or radio (T)
- 51 got their news from print materials such as newspapers or magazines (P)
- 70 got their news from online sources (O)
- 47 got their news from both T and O
- 39 got their news from both T and P
- 36 got their news from both P and O
- 28 got their news from all three T, O, and P

(a) Count and sort these results using a three-set Venn diagram.



100-97 = 3

12+11+4+19+28+8+15

= 97

(b) How many professionals did NOT get their daily news from any of these three sources?

3

9. (8 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

   (a) Donnie and Marie went to the store.

   CONJUNCTION

   (b) When the sun is shining, she gets a sunburn.

   CONDITIONAL

   (c) A number is even if and only if it is divisible by 2.

   BICONDITIONAL

   (d) All dogs bite.

   CONDITIONAL

10. (6 points) Let p = "The dog has rabies." and let q = "The dog bites." Write each statement in words.

   (a) $p \longrightarrow q$

   IF THE DOG HAS RABIES, THEN THE DOG BITES.

   (b) $\sim q \wedge p$

   THE DOG DOES NOT BITE AND THE DOG HAS RABIES.

11. (8 points) Construct the truth table for $(\sim q \wedge p) \longrightarrow p$.

p	q	$\sim q$	$\sim q \wedge p$	$(\sim q \wedge p) \rightarrow p$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

12. (8 points) Consider the following conditional statement:

If tacos are on sale, then today is Tuesday.

(a) State the inverse.

IF TACOS ARE NOT ON SALE, THEN TODAY IS NOT TUESDAY.

(b) State the contrapositive.

IF TODAY IS NOT TUESDAY, THEN TACOS ARE NOT ON SALE.

13. (8 points) Consider the following argument in symbolic form.

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$$

Use your knowledge of common forms of arguments to determine and explain the validity.

THIS ARGUMENT IS VALID.

"ONE OR ANOTHER ($p \vee q$) AND NOT ONE ($\sim p$) INDEED IMPLIES THE OTHER ($\rightarrow q$)"

14. (8 points) Suppose you begin depositing monthly payments into an account earning 7% compounded monthly. Your goal is to accumulate \$18,000 in 8 years. What should your monthly payments be?

$$R = \frac{18000\left(\frac{0.07}{12}\right)}{\left[\left(1+\frac{0.07}{12}\right)^{96}-1\right]} = \$140.41$$

15. (10 points) A couple decides to set aside \$5,000 in a savings account for a second honeymoon. Interest is compounded quarterly at 4.15%.

(a) How much money is in the account after 20 years?

$$A = 5000\left(1 + \frac{0.0415}{4}\right)^{80} = \$11417.67$$

(b) How much money was made in interest?

$$\text{THAT} - 5000 = \$6417.67$$

16. (10 points) A house sells for \$168,850 and an 8% down payment is made. A mortgage is secured for the remaining amount for 30 years at 3.825% compounded monthly.

(a) What amount is financed?

$$168850 - 0.08(168850) = \$155,342.00$$

(b) What is the monthly payment?

$$R = \frac{155342\left(\frac{0.03825}{12}\right)}{\left[1 - \left(1 + \frac{0.03825}{12}\right)^{-360}\right]} = \$726.04$$

(c) When the loan is paid off in 30 years, what will be the total interest paid?

$$360(726.04) - 155342$$

$$= \$106,032.40$$

17. (4 points) Students in a literature class must choose a book to read and a movie to watch. They can choose from 6 different books and 4 different movies. How many different book/movie pairs are there?

$6 \times 4 = \boxed{24}$

18. (4 points) How many 5-person committees can be formed from a group of 18 students?

$${}_{18}C_5 = \frac{18!}{5!\,13!} = \boxed{8568}$$

19. (8 points) A fair six-sided die is rolled.

(a) What is the probability of rolling a 1?

$$\frac{1}{6}$$

(b) What is the probability of rolling a 2 or 4?

$$\frac{2}{6}$$

(c) What are the odds in favor of rolling a 6?

$$\frac{1}{5}$$

(d) What are the odds against rolling a 6?

$$\frac{5}{1}$$

20. (6 points) A woman draws a letter at random from the word *MISSISSIPPI*. If she draws the letter *S*, she wins \$2. If she draws *I*, she wins \$2. If she draws *P*, she wins \$8. And if she draws *M*, she wins \$20. What is the expected value for this game?

Value	Prob
\$2	4/11
\$2	4/11
\$8	2/11
\$20	1/11

$$2\left(\frac{4}{11}\right) + 2\left(\frac{4}{11}\right) + 8\left(\frac{2}{11}\right) + 20\left(\frac{1}{11}\right)$$

$$= \boxed{\frac{52}{11} \approx \$4.73}$$

21. (8 points) In a study of students' homework habits, a professor collected the following data.

	Did homework	Did not do homework	
Received A or B	97	32	129
Received C, D, or F	41	78	119
	138	110	248

A student from this study is selected at random.

(a) What is the probability that the student did homework?

$$\frac{138}{248} \approx 55.6\%$$

(b) What is the probability that the student did homework or received A or B?

$$\frac{138+32}{248} = \frac{170}{248} \approx 68.5\%$$

(c) What is the probability that the student did homework and received A or B?

$$\frac{97}{248} \approx 39.1\%$$

(d) What is the probability that the student received A or B given that he/she did not do homework?

$$\frac{32}{110} \approx 29.1\%$$

22. (8 points) Four letters are selected from the word *MISSISSIPPI* **without replacement**. What is the probability of spelling *SIMP* (in order)?

4/11 S — 4/10 I — 1/9 M — 2/8 P

$$\frac{4}{11} \times \frac{4}{10} \times \frac{1}{9} \times \frac{2}{8} = \frac{32}{7920}$$