Math 112 - Test 2

October 11, 2017

Name __key__

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Which of these sentences are statements? Circle all that apply.

    (a) Jill took a shower today.

    (b) Hurricane Maria caused much damage in Texas.

    (c) Welcome home.

    (d) Please do not use your phone during class.

    (e) 17 is an even number.

    (f) Who is the student body president?

2. (5 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

    (a) You should go to the gas station if you are running low on fuel.

    CONDITIONAL

    (b) When you practice lots of homework problems, you will pass the tests.

    CONDITIONAL

    (c) The figure is a triangle if and only if it is a polygon with three sides.

    BICONDITIONAL

    (d) I am not hungry, but I am going to Jimmy Johns.

    CONJUNCTION

    (e) I'll get paid today or else I am quitting my job.

    DISJUNCTION

3. (3 points) Explain why the sentence "Math 175 is a fun class" is not a statement.

    ↑

    FUN IS SUBJECTIVE.

    THIS IS AN OPINION, AND
    IT TRUTH VALUE DEPENDS
    ON THE OPINION OF THE
    READER. THEREFORE IT DOES
    NOT HAVE A FIXED TRUTH
    VALUE.

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4. (8 points) Write the negation of each statement in a correct sentence.

   (a) His last name has only two letters.

   HIS LAST NAME DOES NOT HAVE TWO LETTERS.

   (b) All dogs are fuzzy.

   SOME DOGS ARE NOT FUZZY.

   (c) Someone in this class will get a B.

   NO ONE IN CLASS WILL GET A B.

   (d) No math student will win the lottery.

   AT LEAST ONE MATH STUDENT WILL WIN THE LOTTERY.

5. (6 points) Let $p =$ "It is cloudy" and let $q =$ "I will go to the beach." Write each statement in words.

   (a) $\sim q \rightarrow p$

   IF I WILL NOT GO TO THE BEACH, THEN IT IS CLOUDY.

   (b) $\sim p \vee q$

   IT IS NOT CLOUDY OR I WILL GO TO THE BEACH.

   (c) $(p \wedge q) \rightarrow p$

   IF IT IS CLOUDY AND I WILL GO TO THE BEACH,
   THEN IT IS CLOUDY.

6. (6 points) Refer to the statements p and q from the problem directly above. Write each statement in symbolic form.

   (a) I will go to the beach if it is cloudy.

   $p \rightarrow q$

   (b) I will not go to the beach if and only if it is cloudy.

   $\sim q \leftrightarrow p$

   (c) It is cloudy and I will go to the beach.

   $p \wedge q$

7. (2 points) If p is false, what is the truth value of $\sim(\sim(\sim p))$?

$$\sim(\sim(\sim F)) = \sim(\sim T) = \sim F = \boxed{T \text{ (TRUE)}}$$

8. (4 points) Without actually constructing it, determine how many rows and columns the truth table for $(p \vee q) \wedge (r \wedge s)$ would have. (Label which answer is which.)
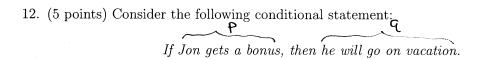
4 STATEMENTS
& 3 OPERATIONS $\longrightarrow$ 7 COLUMNS
16 ROWS

9. (4 points) Write the truth table for $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

TRUE WHEN BOTH p & q HAVE SAME TRUTH VALUE

10. (4 points) Use DeMorgan's Laws to write a logically equivalent statement.

(a) $\sim(p \vee q)$

$$\equiv \quad \sim p \wedge \sim q$$

(b) $\sim(q \wedge r)$

$$\equiv \quad \sim q \vee \sim r$$

11. (4 points) Use DeMorgan's Laws to write the negation of the statement "Sally does not lift weights and she does not run."

SALLY LIFTS WEIGHTS OR SHE RUNS.

NEGATION OF

$\sim p \wedge \sim q$ IS $\sim(\sim p \wedge \sim q)$

$$\equiv p \vee q$$

12. (5 points) Consider the following conditional statement:

$\overbrace{\hspace{2cm}}^{p}$ $\overbrace{\hspace{2cm}}^{q}$

If Jon gets a bonus, then he will go on vacation.

(a) State the inverse. $\sim p \to \sim q$

IF JON DOES NOT GET A BONUS, THEN HE WILL NOT GO ON VACATION.

(b) State the contrapositive. $\sim q \to \sim p$

IF JON WILL NOT GO ON VACATION, THEN HE DOES NOT GET A BONUS.

(c) State the converse. $q \to p$

IF JON WILL GO ON VACATION, THEN HE GETS A BONUS.

(d) Which statement is logically equivalent to the original statement?

Inverse   (Contrapositive)   Converse

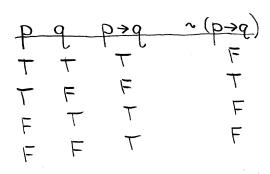(e) Which statement is logically equivalent to the converse?

(Inverse)   Contrapositive   Original

13. (2 points) True or False: The biconditional statement $p \leftrightarrow q$ means the same as $(p \to q) \land (q \to p)$?

TRUE

14. (8 points) Use truth tables to show that the statement $\sim (p \to q)$ is logically equivalent to $\sim q \land p$.

p	q	p→q	~(p→q)
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

p	q	~q	~q∧p
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

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IDENTICAL TRUTH TABLES!
$\sim (p \to q) \equiv \sim q \land p$

8 rows / 6 columns

15. (8 points) Construct the truth table for $p \vee (q \wedge \sim r)$.

p	q	r	$\sim r$	$q \wedge \sim r$	$p \vee (q \wedge \sim r)$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

16. (8 points) By using truth tables, determine whether each statement is a tautology, a self-contradiction, or neither.

(a) $(p \wedge q) \wedge \sim p$

p	q	$p \wedge q$	$\sim p$	$(p \wedge q) \wedge \sim p$
T	T	T	F	F
T	F	F	F	F
F	T	F	T	F
F	F	F	T	F

SELF- CONTRADICTION

(b) $(p \wedge q) \to q$

p	q	$p \wedge q$	$(p \wedge q) \to q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

TAUTOLOGY

17. (4 points) Suppose that the following statement is true:

Deb will eat her sandwich, or she will go hungry.
$\underbrace{\text{Deb will eat her sandwich}}_{p}$, or $\underbrace{\text{she will go hungry}}_{q}$.

What valid conclusion can we draw if $\underbrace{\text{Deb does not go hungry}}_{\sim q}$?

VALID ARGUMENT.
"LAW OF DISJUNCTIVE SYLLOGISM"

$p \vee q$
$\sim q$
$\overline{\therefore \ p}$

WE CONCLUDE THAT DEB ATE HER SANDWICH.

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18. (13 points) Consider the following argument.

p

q

If it is Wednesday, then Sue will eat at *Good Burger*.

It is not Wednesday.

Therefore, Sue will not eat at *Good Burger*.

(a) Write the argument in symbolic form.

$$p \rightarrow q$$
$$\sim p$$
$$\therefore \sim q$$

$$(p \rightarrow q) \wedge \sim p \longrightarrow \sim q$$

4 ROWS

7 COLUMNS

(b) Use the truth table method to determine the validity of the argument.

p	q	$p \rightarrow q$	$\sim p$	$(p \rightarrow q) \wedge \sim p$	$\sim q$	$(p \rightarrow q) \wedge \sim p \rightarrow \sim q$
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F

WE CAN STOP HERE.

NOT A TAUTOLOGY.

Arg. IS NOT VALID

(c) Is the argument a common form? If so, use your knowledge of common forms to explain the validity.

Arg. IS NOT VALID

YES. THIS IS THE FALLACY OF THE INVERSE.

IT IS A FALLACY --- NOT VALID.

$$p \rightarrow q \not\equiv \sim p \rightarrow \sim q$$

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