Math 112 - Final Exam

December 11, 2017

Name ___key___

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Let A be the set of even, one-digit natural numbers.

   (a) Write A in roster notation.

   $$A = \{2, 4, 6, 8\}$$

   (b) What is the cardinality of A?

   $$n(A) = 4$$

   (c) Give an example of a set that is equivalent to A, but not equal to A.

   $$\{a, b, c, d\}$$

2. (4 points) Suppose U (our universal set) is the set of all Prairie State College math students. Let M be the subset of all PSC students taking Math 112. Using words, describe the elements of M'.

   ELEMENTS OF M' ARE PSC STUDENTS

   TAKING A MATH CLASS BUT NOT TAKING MATH 112.

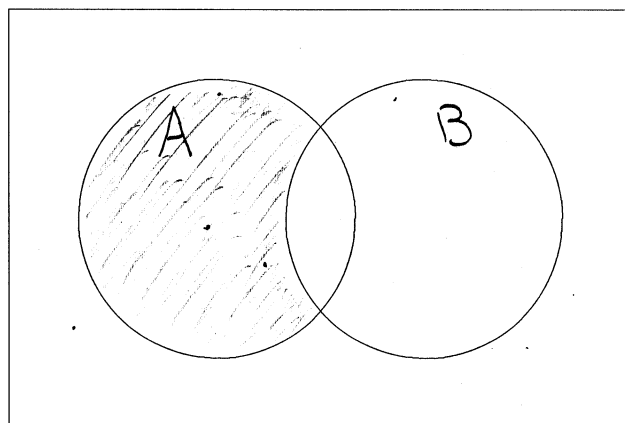3. (4 points) Rewrite each of the following statements using mathematical symbols.

   (a) The number 2 is an element of the intersection of the sets C and D.

   $$2 \in C \cap D$$

   (b) The set of letters of the word *racecar* is a subset of the set M.

   $$\{r, a, c, e\} \subseteq M$$

4. (4 points) In the two-set Venn diagram, shade the region corresponding $A \cap B'$.

↑
A AND NOT B

5. (14 points) Let $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 5, 7, 8\}$, and think of A and B as subsets of the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Determine each of the following.

(a) $n(A) =$ 4

(b) $B' =$ $\{4, 6, 9\}$

(c) $A \cup B =$ $\{1, 2, 3, 4, 5, 6, 7, 8\}$

(d) $A \cap B =$ $\{2, 8\}$

(e) $(A \cup B)' =$ $\{9\}$

(f) $B - A =$ $\{1, 3, 5, 7\}$

(g) $B \cap \emptyset =$ ϕ

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6. (12 points) In a survey of the vegetable preferences of 279 college students, it is found that 68 like Brussels sprouts, 90 like a broccoli, 57 like cauliflower, 29 like both Brussels sprouts and broccoli, 25 like both Brussels sprouts and cauliflower, 24 like both broccoli and cauliflower, and 14 of the students like all three vegetables.

(a) Count and sort these results using a three-set Venn diagram.
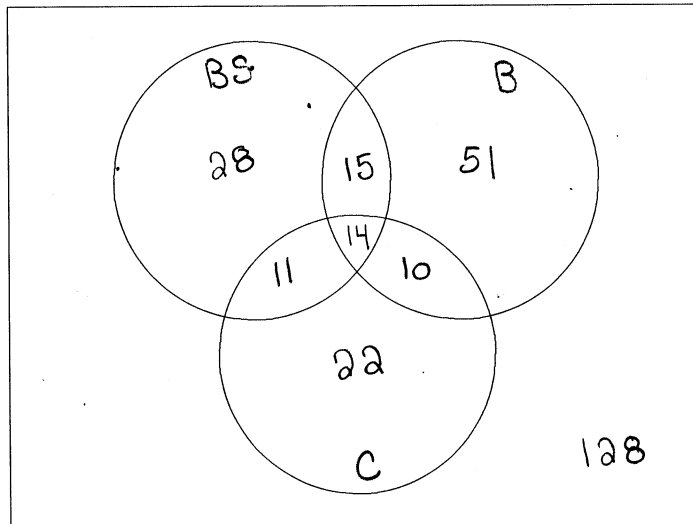
$$28 + 15 + 51 + 11 + 14 + 10 + 22 = 151$$

$$279 - 151 = 128$$

(b) How many students do not like any of these vegetables?

128

(c) How many students like only one of these vegetables?

$$28 + 51 + 22 = 101$$

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7. (8 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

   (a) Either he quits his job, or he gets fired.

   Disjunction

   (b) A number is even if and only if it is divisible by 2.

   Biconditional

   (c) You should wear gloves if your hands are cold.

   Conditional

   (d) It is going to rain or snow.

   Disjunction

8. (6 points) Let $p =$ "The plane lands on time" and let $q =$ "It is snowing." Write each statement in words.

   (a) $q \vee p$

   It is snowing or the plane lands on time.

   (b) $\sim q \longrightarrow p$

   If it is not snowing, then the plane lands on time.

9. (6 points) Refer to the statements p and q from the problem directly above. Write each statement in symbolic form.

   (a) The planes lands on time, but it is snowing.

   $p \wedge q$

   (b) The plane does not land on time whenever it is snowing.

   $q \longrightarrow \sim p$

10. (4 points) Write the negation of the statement "He drinks neither coffee nor tea."

   Not coffee and not tea

   Negation is

   Coffee or tea

   He drinks coffee or tea.

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11. (8 points) Construct the truth table for $(p \vee q) \longrightarrow \sim p$.

p	q	p∨q	~p	(p∨q) → ~p
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

12. (8 points) Consider the following conditional statement:

If Emily graduates, then she will get a job.

(a) State the contrapositive.

IF Emily will NOT get A job, THEN SHE DOES NOT graduate.

(b) State the converse.

IF Emily will get A job, THEN SHE graduates.

(c) Of the following three choices, which one is logically equivalent to the original statement?

Inverse   (Contrapositive)   Converse

13. (8 points) Suppose you begin depositing monthly payments into an account earning 5.5% compounded monthly. Your goal is to accumulate $20,000 in 6 years. What should your monthly payments be?

$$R = \frac{20000 \left(\frac{0.055}{12} \right)}{\left[\left(1 + \frac{0.055}{12} \right)^{(12 \times 6)} - 1 \right]} = \$235.09$$

14. (10 points) A couple sets aside $5,000 in a savings account. Interest is compounded quarterly at 4.05%.

(a) How much money is in the account after 20 years?

$$A = 5000\left(1 + \frac{0.0405}{4}\right)^{(4 \times 20)} = \boxed{\$11,193.^{85}}$$

(b) How much money was made in interest?

$$I = 11193.^{85} - 5000 = \boxed{\$6193.^{85}}$$

15. (12 points) A house sells for $178,000 and an 8% down payment is made. The remaining amount is mortgaged for 30 years at a fixed rate of 4.125% compounded monthly.

(a) What amount is financed?

$$0.92 \times 178000 = \boxed{\$163,760.^{00}}$$

(b) What is the monthly payment?

$$R = \frac{163760\left(\frac{0.04125}{12}\right)}{\left[1 - \left(1 + \frac{0.04125}{12}\right)^{(-12 \times 30)}\right]} = \boxed{\$793.^{66}}$$

(c) When the loan is paid off in 30 years, what will be the total interest paid?

$$12 \times 30 \times 793.^{66} - 163760 = \boxed{\$121,957.^{60}}$$

16. (6 points) At the movies, Shana wants to get snacks for her friends.

(a) How many ways can Shana select three types of candy from eight types of candy offered by the theater?

$$_8C_3 = \frac{8!}{5!\,3!} = \boxed{56}$$

(b) How many ways can Shana select two types of soda from five types of soda offered by the theater?

$$_5C_2 = \frac{5!}{3!\,2!} = \boxed{10}$$

(c) Referring to the problems above, how many ways can Shana select the candy and the soda?

$$56 \times 10 = \boxed{560}$$

17. (6 points) The letters of the English alphabet are used to form a 3-letter passcode.

(a) How many possible codes are there if letters cannot be reused?

$$26 \times 25 \times 24 = \boxed{15,600}$$

(b) How many possible codes are there if letters can be reused?

$$26 \times 26 \times 26 = \boxed{17,576}$$

18. (4 points) Give an example of a probability that might be assigned to

(a) an impossible event

$$0 = 0\%$$

(b) a certain event

$$1 = 100\%$$

(c) an event that is very unlikely, but not impossible

$$0.005 = 0.5\%$$

(d) an event that is 3 times more likely than an event with probability 0.2

$$3 \times 0.2 = 0.6 \quad \text{or} \quad 60\%$$

19. (8 points) A fair six-sided die is rolled.

(a) What is the sample space?

$$\{1, 2, 3, 4, 5, 6\}$$

(b) What is the event of rolling an even? Write the event in roster notation.

$$\{2, 4, 6\}$$

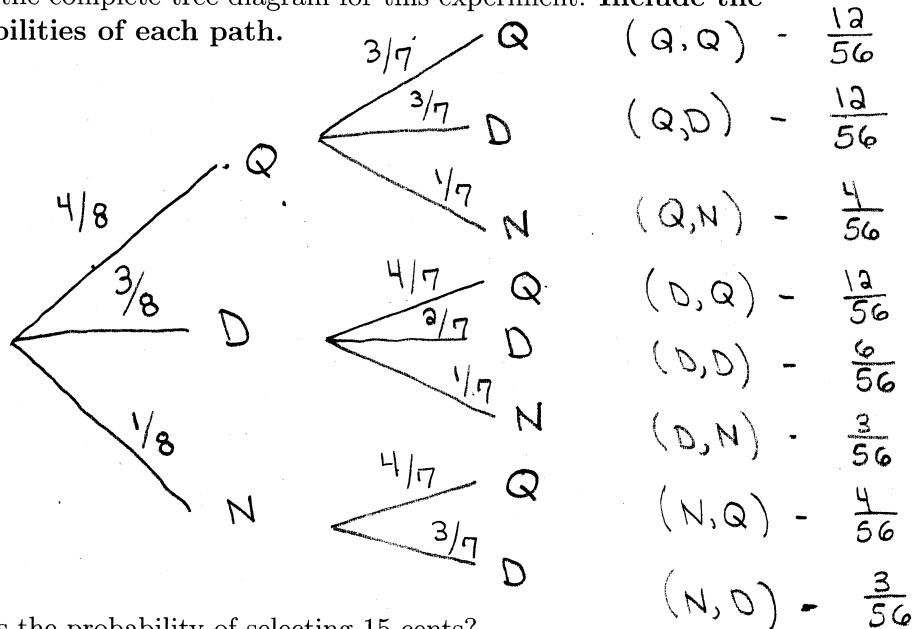(c) What is the probability of rolling a 1?

$$\frac{1}{6}$$

(d) What is the probability of rolling a 2 or 4?

$$\frac{2}{6}$$

20. (12 points) A jar contains 4 quarters, 3 dimes, and 1 nickel. Two coins are selected at random (without replacement).

(a) Sketch the complete tree diagram for this experiment. **Include the probabilities of each path.**

$$3/7 \nearrow Q \qquad (Q,Q) - \frac{12}{56}$$
$$3/7 \rightarrow D \qquad (Q,D) - \frac{12}{56}$$
$$1/7 \searrow N \qquad (Q,N) - \frac{4}{56}$$
$$4/8 \quad Q$$
$$4/7 \nearrow Q \qquad (D,Q) - \frac{12}{56}$$
$$3/8 \quad D \qquad 2/7 \rightarrow D \qquad (D,D) - \frac{6}{56}$$
$$1/7 \searrow N \qquad (D,N) - \frac{3}{56}$$
$$1/8 \quad N$$
$$4/7 \nearrow Q \qquad (N,Q) - \frac{4}{56}$$
$$3/7 \searrow D \qquad (N,D) - \frac{3}{56}$$

(b) What is the probability of selecting 15 cents?

(D,N) or (N,D)

$$\frac{3}{56} + \frac{3}{56} = \boxed{\frac{6}{56}}$$

(c) What is the probability of selecting more than 15 cents?

$$1 - \frac{6}{56} = \boxed{\frac{50}{56}}$$

$\uparrow$ EVERYTHING EXCEPT 15¢.

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