

Math 112 - Test 1
September 12, 2018

Name key Score _____

Show all work. Supply explanations when necessary. Partial credit will be awarded for correct work.

1. (2 points) Let W be the set of letters of the word *banana*. Write W in roster notation.

$$W = \{b, a, n\}$$

2. (2 points) Let X be the set of all natural numbers that are less than 5. Is X well defined? Briefly explain why or why not.

X IS WELL-DEFINED --- WE CAN DETERMINE (OBJECTIVELY) WHAT IS AND IS NOT IN X . IN FACT, $X = \{1, 2, 3, 4\}$.

3. (2 points) Is the set of all funny people well defined? Explain explain why or why not.

IT IS NOT. "FUNNY" IS SUBJECTIVE, NOT OBJECTIVE.
IT IS NOT CLEAR WHO IS "FUNNY".

4. (3 points) The following set is described in set-builder notation. Tell exactly how you would read this description.

$$J = \{x \mid x \in \mathbb{N} \text{ and } x > 100\}$$

"
 J IS THE SET OF ALL x SUCH THAT x
IS A NATURAL NUMBER AND x IS GREATER THAN 100."

5. (4 points) Refer to the problem above. Write J in roster notation and determine $n(J)$.

$$J = \{101, 102, 103, 104, \dots\}$$

$$n(J) = \infty$$

6. (2 points) Give a verbal description for the following set.

$$\{5, 10, 15, 20, 25, \dots\}$$

THE SET OF POSITIVE
MULTIPLES OF 5.

7. (2 points) Write the set {red, white, blue} in set-builder notation.

$$\{x \mid x \text{ IS A COLOR OF THE U.S. FLAG}\}$$

8. (4 points) Think about the set $A = \{\emptyset\}$.

- (a) Determine $n(A)$.

$$n(A) = 1$$

A HAS 1 ELEMENT.

THAT ELEMENT IS THE EMPTY SET.

- (b) Give an example of a set that is equivalent to A , but not equal to A .

$$\{1\} \approx \{\emptyset\}$$

(BOTH SETS HAVE EXACTLY ONE ELEMENT.)

9. (5 points) Decide whether each statement is true or false.

(a) F $18 \notin \{2, 4, 6, 8, 10, 12, \dots\}$ 18 IS IN THE SET

(b) F $0.75 \in \mathbb{N}$ 0.75 IS NOT NATURAL

(c) T $\{x \mid x \text{ is a negative natural number}\}$ is the empty set.

(d) T The set of letters of the word *creeper* has cardinal number 4. $\{c, r, e, p\}$

(e) F $\text{Chicago} \in \{y \mid y \text{ is one of the United States}\}$

CHICAGO IS A CITY,

NOT ONE OF THE STATES.

10. (4 points) Determine the cardinal number for each set.

(a) 100 $A = \{1, 2, 3, 4, 5, \dots, 99, 100\}$

(b) 0 $B = \{ \}$

(c) ∞ $C = \{4, 8, 12, 16, 20, 24, \dots\}$

(d) 1 $D = \{\text{three}\}$

11. (2 points) Give an example of a set that is both equal and equivalent to $\{x, y, z\}$.

THE ONLY POSSIBLE ANSWER IS

$$\{x, y, z\}$$

12. (4 points) Give an example of a single set T that satisfies every one of the following conditions.

• $T \cong \{m, i, s, p\}$

• $T \subseteq \mathbb{N}$

• $17 \in T$

↑ 17 IS ONE
OF THE
ELEMENTS.

← T HAS 4 ELEMENTS

← ELEMENTS ARE NATURAL #'S

$$T = \{1, 2, 3, 17\}$$

13. (4 points) Let A be the set of letters of the word *Mississippi* and let $B = \{1, 2, 3, 4\}$. Use a table to give a pairing of elements that shows that the set A is in a one-to-one correspondence with the set B .

ELEMENT OF A	ELEMENT OF B
m	1
i	2
s	3
3 p	4

14. (1 point) For which one of these sets is it true that $n(V) = 1$?

(a) $V = \emptyset$

(b) $V = \{0, 1\}$

(c) $V = \{1, 11, 111, 1111, \dots\}$

(d) $V = \{\emptyset\}$

15. (1 point) Let $Q = \{a, b, c\}$. Which one of these sets is NOT a proper subset of Q ?

(a) $\{a, b, c\}$

(b) $\{a, b\}$

(c) $\{a\}$

(d) \emptyset

16. (1 point) Which one of the following sets is NOT empty?

(a) $\{ \}$

(b) The set of all natural numbers less than 1

(c) \emptyset

(d) $\{\emptyset\}$

17. (1 point) Which one of the statements below would be read "r is an element of B"?

(a) $r \sim B$

(b) $r \subseteq B$

(c) $r \in B$

(d) $\{r | r \in B\}$

18. (1 point) Let $A = \{1, 2, 3, 4\}$. How many subsets does A have?

(a) 8

(b) 16

(c) 12

(d) 15

$n(A) = 4$

OF SUBSETS = $2^4 = 16$

19. (20 points) Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and think about the subsets $A = \{0, 2, 4, 6, 8\}$ and $B = \{0, 3, 6, 9\}$. Determine each of the following.

(a) $n(A) = 5$

(b) $B' = \{1, 2, 4, 5, 7, 8\}$

(c) $A - B = \{2, 4, 8\}$

(d) $A \cup B = \{0, 2, 3, 4, 6, 8, 9\}$

(e) $A \cap B' = \{0, 6\}$

(f) $(A \cap B)' = \{0, 6\}' = \{1, 2, 3, 4, 5, 7, 8, 9\}$

(g) $A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 2, 4, 5, 7, 8\} = \{1, 2, 3, 4, 5, 7, 8, 9\}$

THESE TWO ARE THE SAME.

(h) $\emptyset \cup B = B = \{0, 3, 6, 9\}$

(i) $A \cap \emptyset = \emptyset$

(j) $A \cup A' = U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

20. (6 points) Suppose the universal set, U , is the set of all current full-time PSC students. Let M be the subset Math 112 students and let E be subset of English 101 students.

(a) Describe, in words, an element of M' .

↳ A Full-time PSC student
NOT TAKING MATH 112

(b) Describe, in words, an element of $M \cap E$.

↳ A Full-time PSC student
TAKING BOTH MATH 112
AND ENGLISH 101.

21. (5 points) List all subsets of the set $\{d, o, g\}$.

$\emptyset, \{d\}, \{o\}, \{g\},$
 $\{d, o\}, \{d, g\}, \{o, g\},$
 $\{d, o, g\}$

THERE ARE 8 OF
THEM.

22. (10 points) Rewrite each of the following statements using mathematical symbols.

(a) The set A is equivalent to the set B .

$$A \cong B$$

(b) The cardinal number of P is 25.

$$n(P) = 25$$

(c) The empty set is a subset of B .

$$\emptyset \subseteq B$$

(d) The number 8 is an element of the set M .

$$8 \in M$$

(e) The set containing only 1 and 2 is a proper subset of the set of natural numbers.

$$\{1, 2\} \subset \mathbb{N}$$

$$A = \{m, i, s, p\}, \quad B = \{a, e, i, o, u\}$$

23. (4 points) Suppose A is the set of letters of the word *Mississippi* and B is the set of vowels in the English alphabet, $\{a, e, i, o, u\}$.

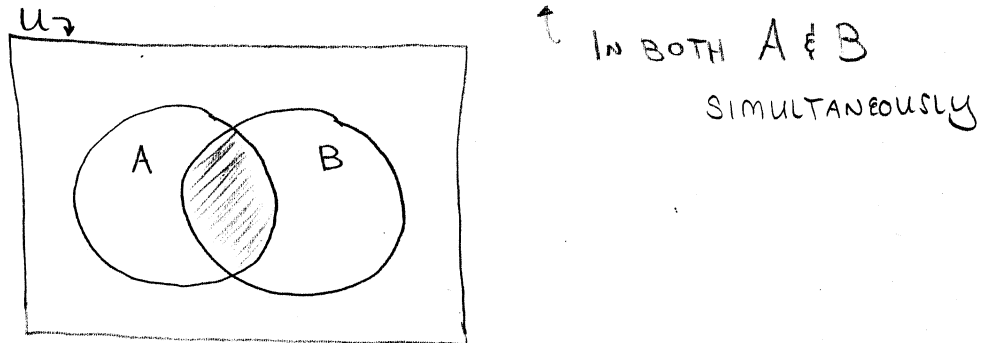
(a) List the elements of $A - B$.

$$A - B = \{m, s, p\}$$

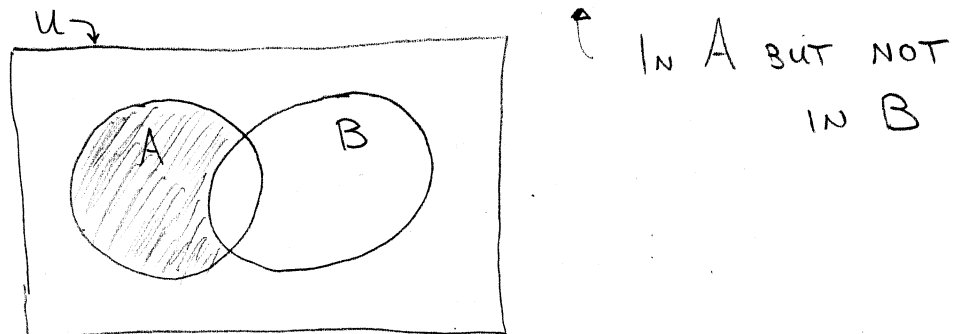
(b) List the elements of $B - A$.

$$B - A = \{a, e, o, u\}$$

24. (4 points) Sketch a Venn diagram that shows two sets, A and B . Label the two sets and then shade the region of the diagram corresponding to $A \cap B$.



25. (4 points) Sketch a Venn diagram that shows two sets, A and B . Label the two sets and then shade the region of the diagram corresponding to $A - B$.



26. (2 points) List all the subsets of $\{1, 2, 3, 4, 5\}$ that are not proper subsets.

THERE IS ONLY ONE!

$$\{1, 2, 3, 4, 5\}$$