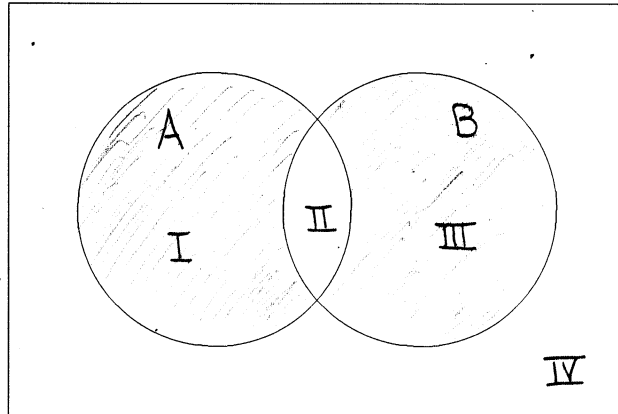


Show all work. Supply explanations when necessary. Partial credit will be awarded for correct work.

1. (6 points) In the two-set Venn diagram shown below, label the sets A and B . Then label the four distinct (disjoint) regions with Roman numerals (or whatever system you prefer to use). Identify and shade the regions that make up $(A \cap B') \cup (A' \cap B)$.



$$A \text{ AND NOT } B = A \cap B' = \{I\}$$

$$B \text{ AND NOT } A = A' \cap B = \{III\}$$

$$(A \cap B') \cup (A' \cap B) = \{I, III\}$$

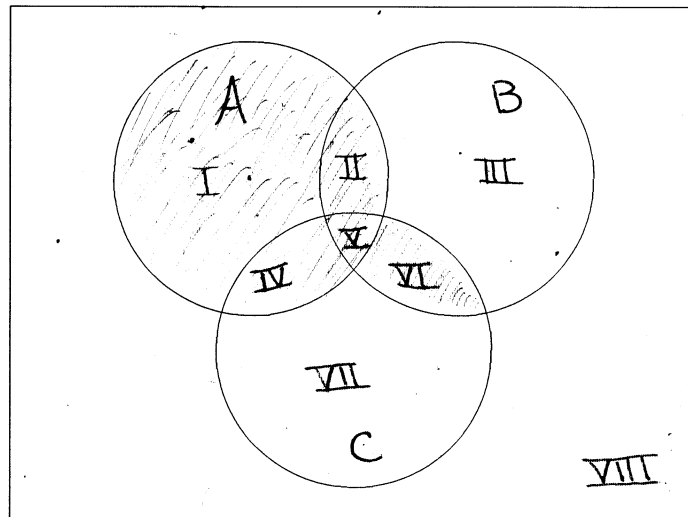
2. (4 points) In class we discussed the formula,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Suppose A has 10 elements, B has 12 elements, and the intersection of A and B has 6 elements. Determine how many elements are in the union of A and B ?

$$n(A \cup B) = 10 + 12 - 6 = \boxed{16}$$

3. (6 points) In the three-set Venn diagram shown below, label the sets A , B , and C . Then label the distinct (disjoint) regions of the diagram with Roman numerals (or whatever system you prefer to use). Identify and shade the regions that make up $A \cup (B \cap C)$.



$$B \cap C = \{V, VI\}$$

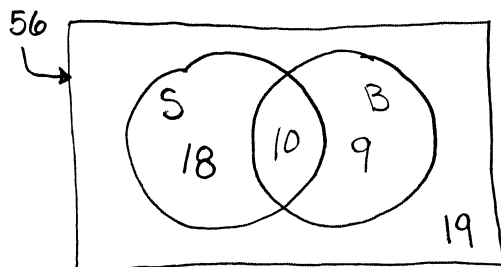
$$A = \{I, II, IV, V\}$$

$$A \cup (B \cap C) = \{I, II, IV, V, VI\}$$

4. (9 points) In a group of 56 high-school athletes, 28 play soccer, 9 play basketball but not soccer, and 10 play both basketball and soccer.

(a) Use a Venn diagram to organize the data.

$$28 - 10 = 18$$



$$18 + 10 + 9 = 37$$

$$56 - 37 = 19$$

(b) How many athletes play soccer or basketball?

$$18 + 10 + 9 = 37$$

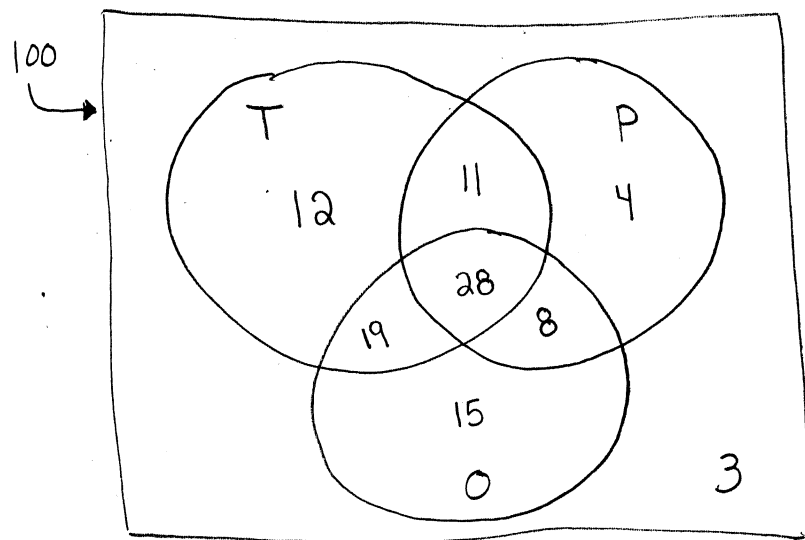
(c) How many athletes play neither soccer nor basketball?

$$56 - 37 = 19$$

5. (12 points) One hundred professionals were asked how they obtained their daily news. Here are the results:

- ✗ 70 got their news from television or radio (T) $11 + 28 + 19 + 12$
- ✗ 51 got their news from print materials such as newspapers or magazines (P) $11 + 28 + 8 + 4$
- ✗ 70 got their news from online sources (O) $19 + 28 + 8 + 15$
- ✗ 47 got their news from both T and O $28 + 19$
- ✗ 39 got their news from both T and P $28 + 11$
- ✗ 36 got their news from both P and O $28 + 8$
- ✗ 28 got their news from all three T, O, and P

- (a) Organize these results using a Venn diagram.



$$12 + 11 + 4 + 19 + 28 + 8 + 15 = 97$$

$$100 - 97 = 3$$

- (b) How many professionals did NOT get their daily news from any of these three sources?

$$100 - 97 = \boxed{3}$$

- (c) How many professionals got their daily news from exactly two of these sources?

$$11 + 19 + 8 = \boxed{38}$$

6. (5 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

(a) You should take a Tylenol if you have a headache.

CONDITIONAL

(b) Either it will rain, or I will cut the lawn.

DISJUNCTION

(c) It is going to rain or snow.

DISJUNCTION

(d) I will go to the store if and only if I need groceries.

BICONDITIONAL

(e) She listened to her iPod, but the battery died.

CONJUNCTION

7. (6 points) Which of these sentences are statements? Circle all that apply.

(a) David Pumpkins is taking Math 112.

(b) Do not run in the hallway.

(c) What time is it?

(d) Terence Tao is a famous mathematician.

(e) $5 + 6 = 56$

(f) Ouch! That hurts!

8. (4 points) Without actually constructing it, determine how many rows and columns the truth table for $(p \vee q) \wedge (r \wedge s)$ would have.

4 STATEMENTS
3 ops

⇒

7 COLUMNS

4 STATEMENTS

⇒

$2^4 = 16$ ROWS

9. (8 points) Write the negation of each statement in a correct sentence.

(a) I do not like to eat mushrooms.

I LIKE TO EAT MUSHROOMS.

(b) No one was late.

SOMEONE WAS LATE.

(c) Every happy dog wags its tail.

SOME HAPPY DOGS DO NOT WAG TAILS.

(d) Some students do not do homework.

ALL STUDENTS DO HOMEWORK.

10. (6 points) Let $p =$ "I have \$5" and let $q =$ "I will not buy pizza." Write each statement in words.

(a) $p \rightarrow \sim q$ IF I HAVE \$5, THEN I WILL BUY PIZZA.

(b) $\sim p \vee q$ I DO NOT HAVE \$5, OR I WILL NOT BUY PIZZA.

(c) $p \leftrightarrow (p \wedge q)$ I HAVE \$5 IF AND ONLY IF I HAVE \$5 AND I WILL NOT BUY PIZZA.

11. (6 points) Refer to the statements p and q from the problem directly above. Write each statement in symbolic form.

(a) I will buy pizza whenever I have \$5.

$p \rightarrow \sim q$

(b) I do not have \$5, but I will buy pizza.

$\sim p \wedge \sim q$

(c) I will buy pizza if and only if I have \$5.

$\sim q \leftrightarrow p$

12. (4 points) Construct the truth table for $(p \vee q) \rightarrow p$.

p	q	$p \vee q$	$(p \vee q) \rightarrow p$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

13. (8 points) Construct the truth table for $p \wedge (q \vee r)$.

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

14. (6 points) Let $p =$ "Today is Saturday" and let $q =$ "Some dogs bite." Determine the truth value of each of the following statements. (You do not need to construct a truth table for each part.)

(a) $p \vee \sim q$ $F \text{ or } F = F$ **FALSE**

(b) $p \rightarrow q$ $F \rightarrow T$ is T **True**

(c) $(p \wedge q) \rightarrow \sim p$
 $(F \text{ AND } T) \rightarrow T$
 $F \rightarrow T$ is T **True**

p is F
 q is T

15. (10 points) By using truth tables, determine whether each statement is a tautology, a self-contradiction, or neither.

(a) $(p \rightarrow q) \vee \sim q$

p	q	$p \rightarrow q$	$\sim q$	$(p \rightarrow q) \vee \sim q$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Tautology \longleftrightarrow All true!

(b) $(p \wedge q) \rightarrow q$

p	q	$p \wedge q$	$(p \wedge q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Tautology \longleftrightarrow All true!