

Math 112 - Final Exam
December 10, 2018

Name key _____
Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) The set W is described in set builder notation as follows:

$$W = \{x \mid x \in \mathbb{N} \text{ and } -3 < x < 6\}.$$

- (a) Write W in roster notation.

$$\{1, 2, 3, 4, 5\}$$

- (b) What is the cardinality of W ?

$$n(W) = 5$$

2. (6 points) Consider the set $A = \{2, 4, 6\}$.

- (a) Give an example of a set B that is equivalent to A , but not equal to A .

$$B = \{a, b, c\}$$

- (b) List all subsets of your set B .

$$\emptyset, \{a\}, \{b\}, \{c\},$$

$$\{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$$

3. (5 points) Tell whether each statement is true (T) or false (F).

(a) F $\frac{1}{2} \in \mathbb{N}$

(b) T $\{3\}$ is equivalent to $\{5\}$

(c) F $\emptyset = \{\emptyset\}$

(d) T $\{1, 3, 5, 7, 9\} \subseteq \{1, 3, 5, 7, \dots\}$

(e) T $9 \in \{x \mid x = 2k + 1 \text{ where } k \in \mathbb{N}\}$

4. (4 points) Give an example of a **single set** D that satisfies **every one** of the following three conditions.

$$D \cong \{a, b, c, d\}, \quad D \subseteq \mathbb{N}, \quad 10 \in D$$

$$D = \{1, 2, 3, 10\}$$

5. (8 points) Let $A = \{11, 13, 15, 17, 19\}$ and $B = \{14, 15, 16, 17\}$, and think of A and B as subsets of the universal set $U = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$. Determine each of the following.

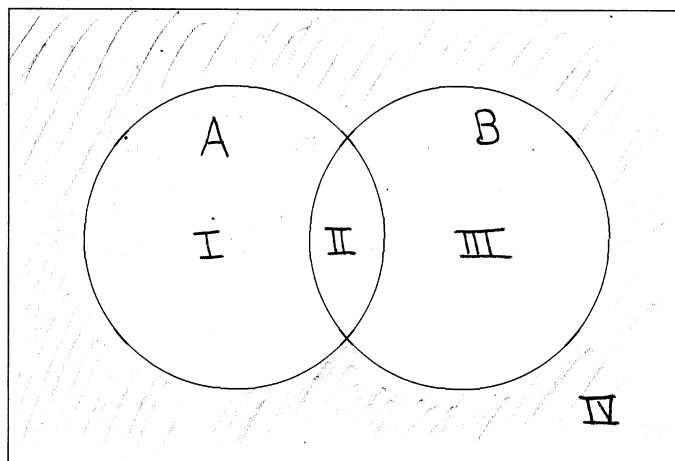
(a) $A' = \{12, 14, 16, 18, 20\}$

(b) $B - A = \{14, 16\}$

(c) $A \cup \emptyset = A = \{11, 13, 15, 17, 19\}$

(d) $A \cap B = \{15, 17\}$

6. (8 points) In the two-set Venn diagram shown below, label the sets A and B . Then label the four distinct (disjoint) regions with Roman numerals (or whatever system you prefer to use). Identify and shade the regions that make up $A' \cap B'$.



$$A' = \{\text{III}, \text{IV}\}$$

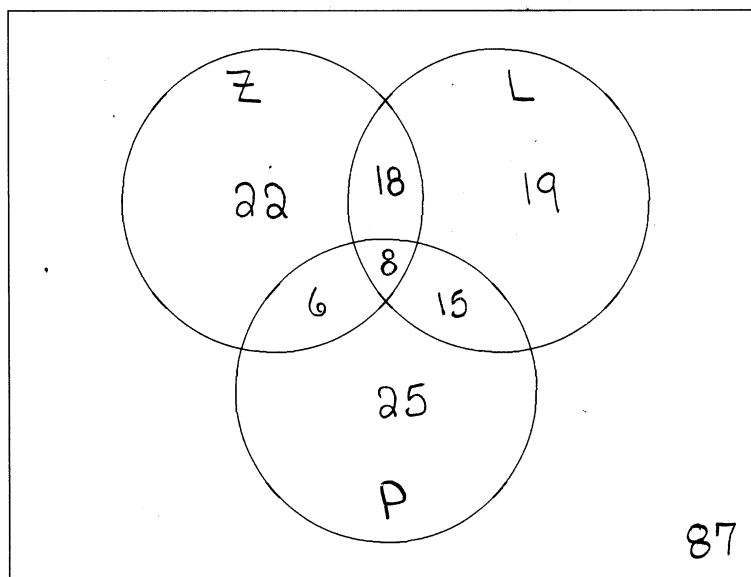
$$B' = \{\text{I}, \text{IV}\}$$

$$A' \cap B' = \{\text{IV}\}$$

7. (14 points) Two hundred patients suffering from depression enrolled in a clinical trial to test the effects of various antidepressants.

- 54 patients were given Zoloft. $54 - 6 - 8 - 18 =$
- 60 patients were given Lexapro. $60 - 18 - 8 - 15 = 19$
- 54 patients were given Prozac. $54 - 6 - 8 - 15 = 25$
- 26 patients were given Zoloft and Lexapro. $\textcircled{26}$
- 23 patients were given Lexapro and Prozac. $\textcircled{23}$
- 14 patients were given Zoloft and Prozac. $\textcircled{14}$
- 8 patients were given all three drugs. $\textcircled{8}$

(a) Count and sort these results using a three-set Venn diagram. (Label your sets!)



$$22 + 18 + 19 + 6 + 8 + 15 + 25 = 113$$

$$200 - 113 = 87$$

(b) How many patients in the trial were given at most two of the three drugs?

ALL BUT MIDDLE 8

$$87 + 22 + 19 + 25 + 18 + 6 + 15 = 192$$

\uparrow NONE ONE TWO

(c) How many patients were treated with Zoloft and Prozac but not Lexapro?

$\boxed{6}$

(d) How many patients were given none of the three drugs?

$\boxed{87}$

8. (5 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

(a) He tried to start his car, but the battery was dead.

CONJUNCTION

(b) You should eat lunch if you are hungry.

CONDITIONAL

(c) Mike went to work, or he went to the grocery store.

DISJUNCTION

(d) I will buy a new phone if and only if my current phone stops working.

BICONDITIONAL

(e) $x^2 = 25$ when $x = 5$.

CONDITIONAL

9. (4 points) Write the negation of each statement.

(a) No dog can run that fast.

SOME DOGS CAN RUN FAST.

(b) Some students write with crayons.

NO STUDENTS WRITE WITH CRAYONS.

10. (6 points) Let $p =$ "The plane is on time." and let $q =$ "The sky is clear." Write each statement in words.

(a) $q \rightarrow p$

IF THE SKY IS CLEAR, THEN THE PLANE IS ON TIME.

(b) $\sim p \vee q$

THE PLANE IS NOT ON TIME, OR THE SKY IS CLEAR.

11. (4 points) Refer to the statements p and q from the problem directly above. Write each statement in symbolic form.

(a) The sky is not clear if the plane is not on time.

$\sim p \rightarrow \sim q$

(b) The plane is on time, or it is not on time.

$p \vee \sim p$

12. (10 points) Use a truth table to determine whether the statement is a tautology, a self-contradiction, or neither.

$$(p \rightarrow q) \vee \sim q$$

p	q	$p \rightarrow q$	$\sim q$	$(p \rightarrow q) \vee \sim q$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

All true \Rightarrow

Tautology

13. (5 points) To train employees to use new equipment, William's Muffler Repair had to borrow \$4500 at 9.5% simple interest. When the company paid off the loan, they ended up paying \$1282.50 in interest. How long was the term of the loan?

$$1282.50 = 4500 (0.095) t$$

$$\Rightarrow t = \frac{1282.50}{(4500 \times 0.095)} = \boxed{3 \text{ yrs}}$$

14. (10 points) Jon deposits \$350 each month into an account earning 6.85% compounded monthly.

- (a) How much money is in the account after 30 years?

$$\frac{350 * \left(\left(1 + \frac{0.0685}{12} \right)^{(12 * 30)} - 1 \right)}{\left(\frac{0.0685}{12} \right)} = \boxed{\$ 414,564.82}$$

- (b) How much money was made in interest?

$$414,564.82 - 350 * 12 * 30 = \boxed{\$ 288,564.82}$$

15. (6 points) How much would you have to invest now into an account earning 2.95% compounded quarterly if you want to have \$6500 in 8 years?

$$6500 = P \left(1 + \frac{0.0295}{4} \right)^{4 \times 8}$$

$$P = \frac{6500}{\left(1 + \frac{0.0295}{4} \right)^{4 \times 8}} = \boxed{\$ 5138.02}$$

16. (16 points) A house sells for \$192,450. You make a 5% down payment and, for the remaining amount, you obtain a 30-year fixed rate mortgage at 4.35% compounded monthly.

- (a) How much is the down payment?

$$0.05 \times 192,450 = \boxed{\$ 9622.50}$$

- (b) What is the loan amount?

$$192,450 - 9622.50 = \boxed{\$ 182,827.50}$$

- (c) What are the monthly payments?

$$\frac{182,827.50 * \left(\frac{0.0435}{12} \right)}{\left(1 - \left(1 + \frac{0.0435}{12} \right)^{-12 * 30} \right)} = \boxed{\$ 910.14}$$

- (d) At the end of the 30-year loan, how much will you have paid in interest?

$$910.14 \times 12 \times 30 - 182,827.50 = \boxed{\$ 144,822.90}$$

- (e) Construct the 1st line of the amortization schedule for the loan. Include the payment number, interest, amount paid to principal, and the outstanding balance.

Payment #	Interest	Amount to Principal	Balance
1	662.75	247.39	182,580.11

$$182,827.50 * (0.0435) * \frac{1}{12} = 662.75$$

17. (5 points) A paint manufacturer plans to make several different paints. The categories include:

Color: Red, blue, white, black, green, brown, yellow 7
Type: Oil, latex 2
Texture: Flat, semigloss, high gloss 3
Use: Outside, inside 2

How many different kinds of paint can be made?

$$7 \times 2 \times 3 \times 2 = \boxed{84}$$

18. (4 points) How many different arrangements can be made from all the letters of the word *TENNESSEE*?

T -- 1
 E -- 4
 N -- 2
 S -- 2

$$\frac{9!}{4!2!2!} = \frac{9 \times 8 \times 7 \times 6 \times 5}{2 \times 1 \times 2 \times 1} = \boxed{3780}$$

19. (8 points) A jar contains one quarter, two dimes, three nickels, and four pennies. A single coin is selected at random.

(a) What is the sample space?

$$\{Q, D, N, P\}$$

(b) What is the event of selecting more than 7 cents? Write the event in roster notation.

$$\{Q, D\}$$

(c) What is the probability of selecting a penny?

$$\boxed{\frac{4}{10}}$$

(d) What is the probability of selecting a coin other than a penny?

$$1 - \frac{4}{10} = \boxed{\frac{6}{10}}$$

20. (4 points) The probability of the event A is $\frac{5}{13}$. What are the odds in favor of A ? What are the odds against A ? (Indicate which answer is which.)

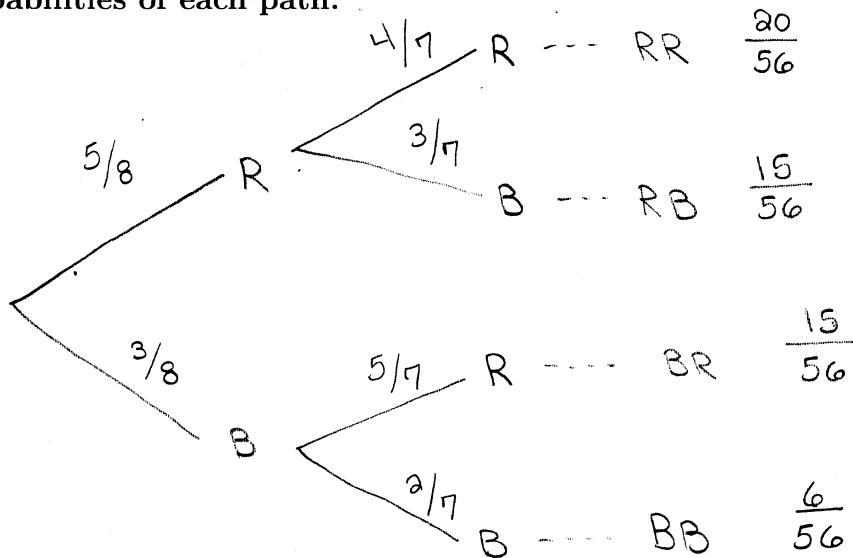
$$13 - 5 = 8$$

$$\text{ODDS IN FAVOR: } \frac{5}{8} = \boxed{5:8}$$

$$\text{ODDS AGAINST: } \frac{8}{5} = \boxed{8:5}$$

21. (12 points) A jar contains 5 red marbles and 3 blue marbles. Two marbles are selected at random, **without replacement**.

- (a) Sketch the complete tree diagram for this experiment. **Include the probabilities of each path.**



- (b) What is the probability of selecting at least one red marble?

$$1 - \frac{6}{56} = \boxed{\frac{50}{56}}$$

- (c) What is the probability of selecting exactly one blue marble?

$$\frac{15}{56} + \frac{15}{56} = \boxed{\frac{30}{56}}$$