Math 112 - Test 1

February 15, 2017

Name __key__

Score _____

Show all work. Supply explanations when necessary. Partial credit will be awarded for correct work.

1. (1 point) Which one of these sets is **NOT** well defined?

   (a) The set of letters in the word *encyclopedia*

   (b) $\{x \mid 2x + 1 = 5\}$

   (c) The set of all beautiful flowers   NOT CLEAR WHAT FLOWERS ARE BEAUTIFUL.

   (d) $\{1, 3, 5, 6, 8, 10\}$

2. (1 point) Which one of the sets given below is **NOT** equivalent to $\{5\}$?

   (a) $\{5\}$

   (b) $\{\emptyset\}$

   (c) $\{0\}$

   (d) $\emptyset$ ← ONLY SET IN LIST THAT DOES NOT HAVE EXACTLY ONE ELEMENT

3. (1 point) Let $M = \{1, 2, 3\}$. Which one of these sets is a proper subset of M?

   (a) $\emptyset$ ↳ SUBSET, BUT NOT EQUAL TO

   (b) $\{1, 2, 3\}$

   (c) $\{a, b, c\}$

   (d) $\{1, 3, 5\}$

4. (1 point) For which one of the sets defined below is it true that $n(A) = 4$?

   (a) A is the set of of all solutions of the equation $2x = 8$.

   (b) $A = \{0, 1, 2, 3, 4\}$

   (c) A is the set of letters of the word *Mississippi*. $A = \{m, i, s, p\}$

   (d) A is the set of letters of the word *noon*.

5. (1 point) Let $U = \{1, 2, 3\}$. How many subsets does U have?

   (a) 3

   (b) 6

   (c) 7

   (d) 8

   $\{1, 2, 3\}$

   $\{1, 2\}, \{1, 3\}, \{2, 3\}$

   $\{1\}, \{2\}, \{3\}$

   $\emptyset$

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6. (5 points) Determine whether each statement is true or false.

(a) __T__ For sets X and Y, the statement $X = Y$ means the same as $X \subseteq Y$ **and $Y \subseteq X$**.

(b) __F__ The set $A = \{1, 2, 3, 4, 5, 6, 7, 8, \ldots\}$ has 8 elements. $n(A) = \infty$

(c) __T__ All equal sets are equivalent.

EQUIVALENT MEANS
IN ONE-TO-ONE CORRESPONDENCE

(d) __F__ All equivalent sets are equal.

(e) __F__ $n(\{\emptyset\}) = 0$.

↳ $\{\phi\}$ HAS 1 ELEMENT

7. (5 points) Rewrite each of the following statements using mathematical symbols.

(a) The number 2 is an element of the set U.

$$2 \in U$$

(b) The set D is a subset of the set E.

$$D \subseteq E$$

(c) The cardinality of the set Q is infinity.

$$n(Q) = \infty$$

(d) The number -1 is not an element of the set of natural numbers.

$$-1 \notin \mathbb{N}$$

(e) The empty set is not equal to the set whose only element is the empty set.

$$\phi \neq \{\phi\}$$

8. (4 points) Rewrite the set S in roster notation.

$$S = \{x \mid x \in \mathbb{N} \text{ and } -2 \leq x < 4\}$$

$$= \{1, 2, 3\}$$

9. (6 points) Let B be the set of one-digit natural numbers.

   (a) Write B in roster notation.
$$B = \{1,2,3,4,5,6,7,8,9\}$$

   (b) What is the cardinality of B?
$$n(B) = 9$$

   (c) Give an example of a set that is equivalent to B, but not equal to B.
$$\{a,b,c,d,e,f,g,h,i\}$$

10. (4 points) Suppose U is the set of all people who have won the Nobel Peace Prize. Let A be the subset of all American Nobel Peace Prize winners. Using words, describe the elements of A'.

A' IS THE SET OF ALL NOBEL PEACE

PRIZE WINNERS WHO ARE NOT AMERICAN.

11. (6 points) Let $A = \{4, 6, 8, 10\}$ and $B = \{2, 6, 12\}$.

   (a) Determine the difference $A - B$.

$$A - B = \{4, 8, 10\}$$

   (b) Determine the difference $B - \emptyset$.

$$B - \emptyset = B = \{2, 6, 12\}$$

   (c) How many elements are in the Cartesian product $B \times A$?

$$B \times A = \{(2,4), (2,6), (2,8), (2,10),$$
$$\cdots$$
$$(12, 10)\}$$

$$n(B \times A) = 3 \cdot 4 = \boxed{12}$$

12. (4 points) List all subsets of $\{1, 2\}$.

$\{1,2\}, \quad \{1\}, \{2\}, \phi$

13. (16 points) Let $A = \{0, 1, 2, 3\}$ and $B = \{0, 1, 3, 5, 9\}$, and think of A and B as subsets of the universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Determine each of the following.

(a) $n(A)$ $= 4$

(b) $B' = \{2, 4, 6, 7, 8\}$

(c) $A \cup B = \{0, 1, 2, 3, 5, 9\}$

(d) $A \cap B = \{0, 1, 3\}$

(e) $(A \cap B)' = \{2, 4, 5, 6, 7, 8, 9\}$

(f) $A' \cup B' = $ SAME AS $\uparrow$ SINCE $(A \cap B)' = A' \cup B'$

(g) $B \cap \emptyset = \phi$

(h) $A \cup \emptyset = A = \{0, 1, 2, 3\}$

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14. (2 points) Let $C = \{\text{red}, \text{black}, \text{blue}\}$ and $M = \{\text{Ford}, \text{Chevy}\}$. Determine the Cartesian product $C \times M$.

$$C \times M = \{ (\text{red, Ford}), (\text{red, Chevy}),$$
$$(\text{black, Ford}), (\text{black, Chevy}),$$
$$(\text{blue, Ford}), (\text{blue, Chevy}) \}$$

15. (5 points) Let $A = \{0, 2, 4, 6, 8\}$ and let $B = \{1, 2, 3, 4, 5\}$. Determine whether each statement below is true or false.
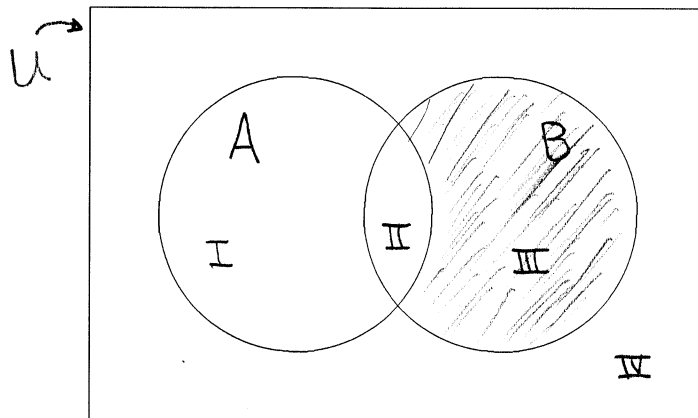
(a) $6 \in (A \cup B)$ · True

(b) $n(A \cap B) = 8$ False $A \cap B = \{2, 4\}$

(c) $5 \subseteq B$ False $5 \in B$

(d) $5 \in A$ False

(e) $\emptyset \subseteq A$ True, THE EMPTY SET IS A SUBSET OF EVERY SET

16. (4 points) In the two-set Venn diagram, shade the region corresponding $A' \cap B$. Show work or explain your reasoning.
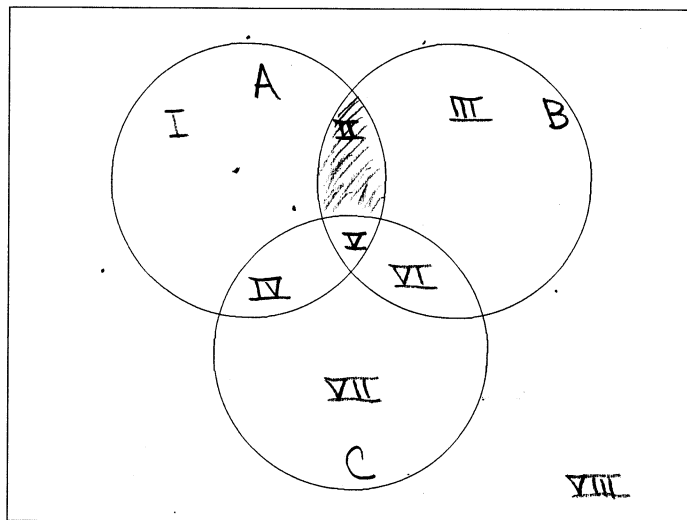

$A = \{ I, II \}$

$A' = \{ III, IV \}$

$B = \{ II, III \}$

$A' \cap B = \{ III \}$

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17. (10 points) Shade the region corresponding to each set. Show work or explain your reasoning.
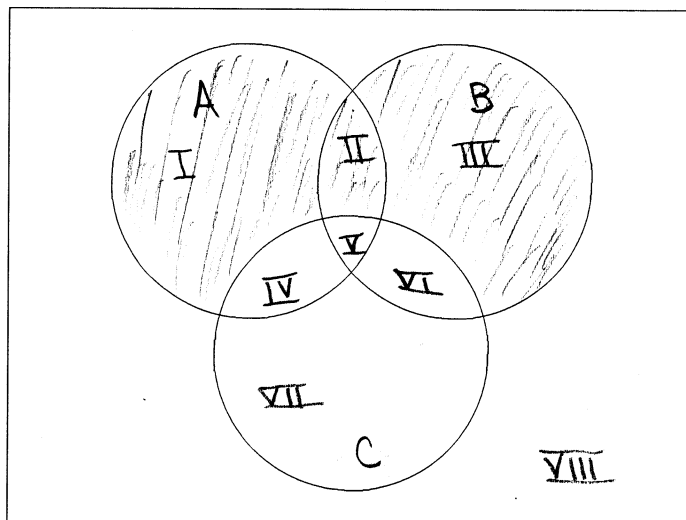
(a) $(A \cap B) \cap C'$

IN A AND IN B
BUT NOT
IN C

$A \cap B = \{ II, V \}$

$C' = \{ I, II, III, VIII \}$

$(A \cap B) \cap C'$
$= \{ II \}$

(b) $(A \cup B) - C$

COMBINED
A & B WITH
C REMOVED

$A \cup B =$
$\{ I, II, III, IV, V, VI \}$

REMOVE THOSE
IN C

TO get

$(A \cup B) - C$
$= \{ I, II, III \}$

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The next two problems are take-home problems. They are due Monday, February 20, with no exceptions.

18. (8 points) The following information was obtained from a survey of 64 MS-Windows computer users.

   - 29 use Notepad
   - 21 use Wordpad
   - 22 use neither Notepad nor Wordpad

(a) Sort and arrange the information in an appropriate Venn diagram.
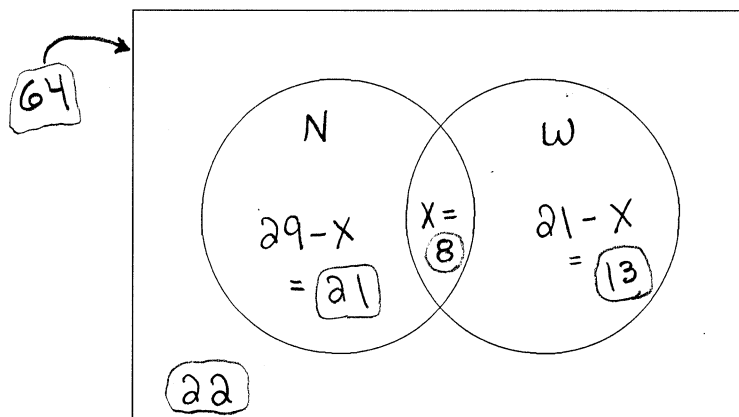
$$(29-x) + x + (21-x) + 22 = 64$$

$$72 - x = 64 \Rightarrow x = 8$$

(b) How many of the people surveyed use both Notepad and Wordpad?

$$\boxed{8}$$

(c) How many of the people surveyed use only Notepad or only Wordpad (not both)?

$$21 + 13 = \boxed{34}$$

19. (12 points) Mr. Strand asked 86 students at his school about the kinds of fruit they normally eat. Here is what he found:

- 43 eat apples
- 36 eat mangoes
- 33 eat kiwis
- 13 eat kiwis and mangoes
- 15 eat apples and mangoes
- 16 eat kiwis and apples
- 8 eat all three

(a) Count and sort these results using a three-set Venn diagram.

86

A
20   7   M
          8   16
     8   5
        12
        K
10
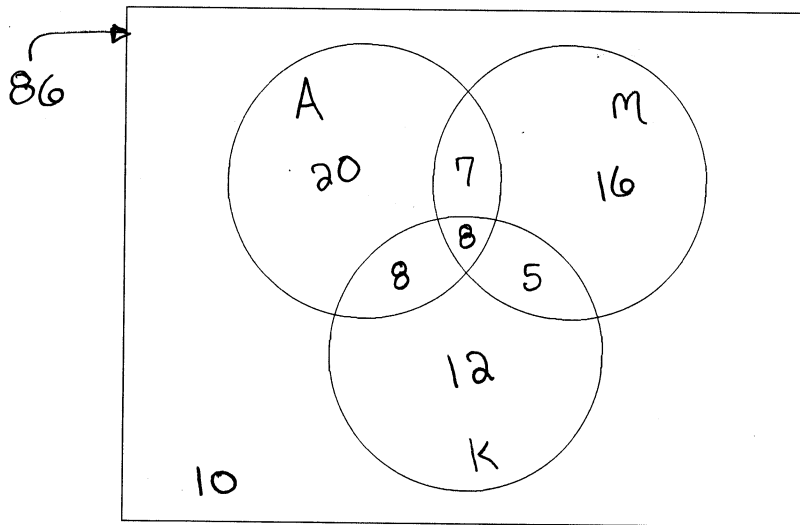
$20 + 7 + 16 + 8 + 8 + 5 + 12$
$= 76$

$86 - 76 = 10$

(b) How many students do not eat any of these fruits?

$\boxed{10}$

(c) How many students eat two of the fruits, but not all three?

$7 + 8 + 5 = \boxed{20}$

(d) How many students eat only one kind of fruit?

$20 + 16 + 12 = \boxed{48}$

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