Name __key__

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (7 points) Consider the following conditional statement:

    If the stock is cheap, then many people buy it.

    (a) Write the contrapositive as a complete sentence.

    IF MANY PEOPLE DO NOT BUY IT, THEN THE STOCK IS NOT CHEAP.

    (b) Write the inverse as a complete sentence.

    IF THE STOCK IS NOT CHEAP, THEN MANY PEOPLE DO NOT BUY IT.

    (c) Write the converse as a complete sentence.

    IF MANY PEOPLE BUY IT, THEN THE STOCK IS CHEAP.

    (d) Of the three, which is equivalent to the original statement?

    Inverse   (Contrapositive)   Converse

2. (3 points) Suppose that the following conditional statement is true:

    Whenever Mary goes shopping, she spends lots of money.

    What, if anything, can we conclude if Mary does not spend lots of money?

    THE ORIGINAL (TRUE) STATEMENT SAYS

    "IF MARY GOES SHOPPING, THE SHE SPENDS LOTS OF MONEY."

    THIS IS LOGICALLY EQUIVALENT TO ITS CONTRAPOSITIVE:

    "IF SHE DOES NOT SPEND LOTS, THEN SHE DOES NOT GO SHOPPING."

    WE CONCLUDE THAT (SHE DOES NOT GO SHOPPING.)

3. (8 points) Consider the following argument.

$\overbrace{\text{If you work hard}}^{P}$, then $\overbrace{\text{you will become wealthy}}^{q}$.
If you become wealthy, then you can become President.
_____
Conclusion: If you work hard, then $\underbrace{\text{you can become President}}_{r}$.

(a) Write the argument in symbolic form.

$$p \to q$$
$$q \to r$$
$$\overline{\therefore \ p \to r}$$

(b) The argument is a common form. What common form is it? Is the argument valid?

THIS IS A TRANSITIVITY ARGUMENT.

IT IS VALID.

4. (6 points) Mario deposits $500 at the end of each quarter into an account earning 8% compounded quarterly. How much money will be in the account after 22 years?

ANNUITY!

$R = 500$
$r = 0.08$
$n = 4$
$t = 22$

$A = ?$

$$A = \frac{500\left(\left(1 + \frac{0.08}{4}\right)^{(4 \times 22)} - 1\right)}{\left(\frac{0.08}{4}\right)} =$$

$\$ 117,808.85$

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5. (14 points) Consider the following argument.

$$\overbrace{\text{I studied}}^{p} \text{ or } \overbrace{\text{I failed the class.}}^{q}$$

I did not fail the class.

Therefore, I studied.

(a) Write the argument in symbolic form.

$$p \lor q$$
$$\sim q$$
$$\overline{}$$
$$\therefore p$$

(b) Use the truth table method to determine the validity of the argument.

$$[(p \lor q) \land (\sim q)] \longrightarrow p$$

p	q	p∨q	∼q	(p∨q)∧∼q	[ ] → p
T	T	T	F	F	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	F	T	F	T

TAUTOLOGY
⇒ VALID

(c) Does this argument have a common form? If so, use your knowledge of common forms to explain the validity.

IT IS A COMMON FORM:

THE LAW OF DISJUNCTIVE
SYLLOGISM, BUT WE CALLED
IT "ONE OR THE OTHER AND NOT ONE"

THIS COMMON ARGUMENT
IS VALID.

6. (6 points) Tonya deposited $1200 into an investment account earning 5.35% simple interest. At the end of the investment period, she had $1504.95. How long was the investment period?

$$I = 1504.95 - 1200 = 304.95$$

$$304.95 = (1200)(0.0535)\, t$$

$$\Rightarrow t = \frac{304.95}{(1200)(0.0535)} = \boxed{4.75 \text{ yrs}}$$

3

7. (18 points) A house sells for \$178,850. You make a 9% down payment. For the remaining amount, you obtain a 30-year, fixed-rate mortgage at 4.25% compounded monthly.

(a) What is the amount of the down payment?

$$9\% \text{ of } 178850$$

$$= (0.09)(178850)$$

$$= \boxed{\$16,096.^{50}}$$

(b) What is the loan amount?

$$178,850 - 16096.5 = \boxed{\$162,753.^{50}}$$

(c) What are your monthly payments on the loan?

$$R = \frac{(162753.50)(0.0425/12)}{\left(1 - \left(1 + 0.0425/12\right)^{\wedge(-12\times30)}\right)}$$

$$= \boxed{\$800.^{65}}$$

(d) How much in total will you end up paying toward the mortgage?

$$12 \times 30 \times 800.65 = \boxed{\$288,234.^{00}}$$

(e) At the end of the 30-year term, how much interest will you have paid?

$$288234 - 162753.50 = \boxed{\$125,480.^{50}}$$

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8. (6 points) Samuel needs $2125 to purchase office equipment. He gets a loan at 9% simple interest for three years.

(a) How much interest will Samuel pay?

$$I = (2125)(0.09)(3) = \boxed{\$573.75}$$

(b) Samuel decides to pay the total amount (principal + interest) in 36 equal monthly payments. How much is each payment?

$$\frac{2125 + 573.75}{36} = \boxed{\$74.97}$$

9. (8 points) Determine the better investment: 3% compounded daily or 3.1% compounded semiannually. (Compute and compare the effective interest rates.)

3% DAILY

$$E = \left(1 + \frac{0.03}{365}\right)^{365} - 1$$

$$= 0.03045$$

$$= \boxed{3.045\%}$$

3.1% SEMI

$$E = \left(1 + \frac{0.031}{2}\right)^{2} - 1$$

$$= 0.03124$$

$$= \boxed{3.124\%}$$

☆

3.1%
SEMI IS
BETTER

10. (10 points) A 24-year-old plans to retire at age 60. She decides to invest an inheritance of $65,000 at 7.5% compounded quarterly.

(a) How much money will be in the account when she is 60 years old?

$60 - 24 = 36$ years

$$A = 65000 \left(1 + \frac{0.075}{4}\right)^{(4 \times 36)}$$

$$= \boxed{\$943,302.^{29}}$$

(b) How much money is made in interest?

$$943302.^{29} - 65000$$

$$= \boxed{\$878,302.^{29}}$$

11. (8 points) A company needs to have $2,000,000 in 8 years. The company will make semiannual payments into an account earning 7.75% compounded semiannually. How much will the semiannual payments be on the annuity?

$R = ?$
$r = 0.0775$
$n = 2$
$t = 8$
$A = 2000000$

$$R = \frac{2000000 \left(\frac{0.0775}{2}\right)}{\left(\left(1 + \frac{0.0775}{2}\right)^{(2 \times 8)} - 1\right)}$$

$$= \boxed{\$92,561.^{05}}$$

12. (6 points) How much should be deposited now into an account earning 6.35% compunded monthly in order to have $10,000 in 12 years?

$$10000 = P \left(1 + \frac{0.0635}{12}\right)^{(12 \times 12)}$$

$$10000 = P \times 2.13825\ldots$$

$$\boxed{P = \$4676.^{71}}$$

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